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ON THE NONLINEARITIES OF FLUID FLOW IN NONRIGID TUBES

BY

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ABSTRACT

Several discrepancies between linear theory and experimental results are explained by the nonlinear theory. Two simultaneous nonlinear differential equations are considered for the non-steady one-dimensional motion of an ideal fluid in a rotationally symmetric tube. To these must be added an equation of state of the tube expressing the tube radius as a function of the pressure difference. The general solution of these equations for a linearly elastic tube is expressed in an analytic form convenient for computation on a digital computer. A graphic method of solution is also presented and applied to the problem of a large pulse wave in a semi-infinite tube. Data for the problem presented are intended to simulate the problem of a heart pulse wave in the aorta. Exact stress-strain data for the aorta and approximate heart pulse data are used. The effect of the nonlinearities included is found to be significant.

INTRODUCTION

Essentially, all of the work on fluid filled elastic tubes which has appeared in the literature has been linearized (7, 8, 12).[2] This paper attempts to take into account some of what are felt to be the more significant nonlinear effects, namely, the convective terms in the equation of motion and the nonlinearities of the stress-strain curve of human tissue (9) when subject to a large deformation.

ASSUMPTIONS

1. The fluid is assumed incompressible. It is felt that the compressibility of blood is insignificant as compared to the distensibility of the soft arterial walls.

2. The fluid is assumed to be nonviscous.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

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3. The mass of the material wall was neglected. It is felt that the effect of the mass of the wall would be small compared to the effect of the mass of the blood.

4. The axial stress in the wall is neglected. This assumption is somewhat more controversial than the previous ones. Primary reason for neglecting it was that it will contribute a higher order derivative to the equations as well as introducing an additional dependent variable, namely, the axial displacement of the tube itself. The secondary reason is that, if the tube is long, the effect of axial stress will be less significant. The mathematical effect on the model used is to permit discontinuous solutions which can be discussed in a manner analogous to the theory of shock waves in the flow of compressible fluids.

5. The model discussed is a one-dimensional one, that is, variations of the quantities in an axial direction only are considered. The radial variations are not completely neglected, however; as shown in (6), the equations used result from a perturbation of the two-dimensional axially symmetric equations. Thus it might be said that the radial effects are systematically linearized. This approximation is very similar to the one used in nonlinear shallow water theory (11).

6. The flow is assumed to be rotationally symmetric.

7. Bending stress in the tube is neglected.

Under the foregoing assumptions, the differential equations simplify to two simultaneous nonlinear partial differential equations for the velocity, pressure, and tube radius which are functions of axial position along the tube and the time. One of these equations results from the conservation of momentum, and the other from the conservation of mass. To these must be added an equation of state of the tube giving the radius of the tube as a function of pressure.

The general equations of momentum and continuity are:

$$\begin{aligned} 2\eta \frac{\partial \xi}{\partial x} + \xi \frac{\partial \eta}{\partial x} + 2\frac{\partial \xi}{\partial \tau} &= 0 \\ \frac{\partial \eta}{\partial \tau} + \eta \frac{\partial \eta}{\partial x} + \frac{\partial P}{\partial x} &= 0 \end{aligned} \tag{1}$$

where

η = dimensionless fluid velocity $= \dfrac{u}{\sqrt{\alpha}}$

ξ = dimensionless tube radius $= \dfrac{R}{R_0}$

x = dimensionless position along the tube axis $= \dfrac{\bar{x}}{L_0}$

τ = dimensionless time $= \dfrac{t\sqrt{\alpha}}{L_0}$

P = dimensionless fluid pressure $= \dfrac{\bar{P}}{\rho\alpha}$

$\alpha = \dfrac{Eh}{\rho R_0}$

$R = R(x, t)$ = radius of the tube

R_0 = unstressed radius

$\bar{x}$ = axial position along tube axis

L_0 = a characteristic length

u = mean fluid velocity

t = time in seconds

$\bar{P}$ = fluid pressure

ρ = mass density of fluid

E = modulus of elasticity of tube in tangential direction

h = tube thickness.

The derivations of these equations were considered too lengthy to be included in this paper. They may be found in (6).

We assume now that the radius of the tube is uniquely determined by the pressure difference across the tube wall, thus, $\xi = \xi(P)$, and inversely:

$$P = P(\xi). \tag{2}$$

We now substitute (2) into (1) and solve for the characteristics of Eqs. 1. The characteristics of a differential equation are open to many interpretations. Physically, when the characteristics are real (hyperbolic case) they represent families of lines in the $x - \tau$ or physical plane along which discontinuities, originating on the boundary, can be propagated. Mathematical derivations and methods of calculation may be found in (2) or (10). The characteristics of Eqs. 1 are defined by the differential equations

$$\frac{dx}{d\tau} = \eta \pm \sqrt{\frac{\xi}{2}\frac{dp}{d\xi}}. \tag{3}$$

Since an increase in fluid pressure increases the radius of the tube, and since the radius is always positive, the quantity under the radical is always positive, and hence Eqs. 1 are always hyperbolic. We note further that Eq. 3 represents the velocities at which disturbances are propagated along the tube. In general, these velocities depend on the solution itself. One velocity is always positive and the other may be positive or negative. It is interesting, also, at this point to introduce a particular equation of state. Considering the equilibrium of a tube of linear elasticity cut by a diametral plane, we have

$$(\bar{P} - \bar{P}_e)2R = 2Eh\frac{R - R_0}{R_0}. \tag{4}$$

The dimensionless form of (4) is

$$P = P_e + 1 - \frac{1}{\xi}, \tag{5}$$

and from (5),

$$\frac{dP}{d\xi} = \frac{1}{\xi^2}. \tag{6}$$

Inserting (6) into (3) and considering a linearized case with fixed values of ξ and η, we have

$$\frac{dx}{d\tau} = \eta_0 \pm \sqrt{\frac{1}{2\xi_0}}. \tag{7}$$

Putting (7) into dimensional form, we have

$$\frac{d\bar{x}}{dt} = U_0 \pm \sqrt{\frac{Eh}{2\rho R_0}}. \tag{8}$$

This can be recognized as the usual expression for the velocities of small disturbances in a fluid filled elastic tube (13). It was, however, only recently (8) shown that the average fluid velocity U_0 should properly appear.

Returning now to the general Eq. 1 together with the particular equation of state (5) we note that they are linear and homogeneous with respect to the first partial derivatives and, hence, they may be linearized with the so-called hodograph (Legendre) transformation. This results in the equations

$$\begin{aligned} \xi^2 \frac{\partial x}{\partial \xi} - \xi^2 \eta \frac{\partial \tau}{\partial \xi} + \frac{\partial \tau}{\partial \eta} &= 0 \\ 2\eta \frac{\partial \tau}{\partial \eta} - \xi \frac{\partial \tau}{\partial \xi} - 2\frac{\partial x}{\partial \eta} &= 0. \end{aligned} \tag{9}$$

This transformation is valid if and only if the Jacobian

$$J = \frac{\partial \xi}{\partial x}\frac{\partial \eta}{\partial \tau} - \frac{\partial \xi}{\partial \tau}\frac{\partial \eta}{\partial x} \neq 0. \tag{10}$$

The characteristics of Eqs. 9 are given by

$$\frac{d\xi}{d\eta} = \pm \frac{\xi^{3/2}}{\sqrt{2}}. \tag{11}$$

We point out for later use that the general form of (11) is

$$\frac{d\eta}{d\xi} = \pm \sqrt{\frac{2}{\xi}\frac{dP}{d\xi}}. \tag{12}$$

GENERAL SOLUTION

We shall present now a general solution for Eqs. 1 subject to the restrictions that the hodograph transformation for both Eqs. 1 and for the boundary conditions exist and, further, that the tube state is adequately described by (5).

Equation 11 defines two fixed sets of curves in the $\xi - \eta$ plane which are independent of the solutions $\xi(x, \tau)$; $\eta(x, \tau)$.

Integration of Eqs. 11 gives

$$\begin{aligned} \eta - 4\sqrt{\frac{1}{2\xi}} &= \text{const} = \sigma \\ \eta + 4\sqrt{\frac{1}{2\xi}} &= \text{const} = \mu \end{aligned} \tag{13}$$

where σ = constant or μ = constant gives the two families of characteristic curves. σ and μ may be interpreted as characteristic coordinates.

Writing Eq. 9 in terms of the characteristic coordinates and eliminating x by differentiation we have

$$\frac{\partial^2 \tau}{\partial \mu \partial \sigma} + \frac{3}{2(\mu + \sigma)}\left(\frac{\partial \tau}{\partial \mu} - \frac{\partial \tau}{\partial \sigma}\right) = 0. \tag{14}$$

Comparison of (14), with the similar expression for one-dimensional compressible flow, shows that (14) is the same as that obtained for a fluid with a ratio of specific heats of 1/2; however, no real analogy can be found. We now change variables by letting

$$\begin{aligned} \mu + \sigma &= u (= 2\eta \text{ from Eq. 13}) \\ \mu - \sigma &= v \left(= 8\sqrt{\frac{\xi}{2}} \text{ from Eq. 13}\right). \end{aligned} \tag{15}$$

Substitution of (15) into (14) yields

$$\frac{\partial^2 \tau}{\partial u^2} + \frac{\partial^2 \tau}{\partial v^2} + \frac{3 \partial \tau}{v \partial v} = 0. \tag{16}$$

Now assume that τ is of the form

$$\tau = U(u) \cdot V(v). \tag{17}$$

Substitution of (17) into (16) yields

$$\frac{V''}{V} - \frac{3}{v} \cdot \frac{V'}{V} = \frac{U''}{U} = -k^2. \tag{18}$$

Solutions of (18) are

$$U = C_1 \sin(ku + C_2)$$
$$V = C_3 v^2 J_2(kv) + C_4 v^2 Y_2(kv) \tag{19}$$

where J_2 and Y_2 are second order Bessel Functions of the first and second kind. Substitution of (15) into (19) and then (19) into (17) gives

$$\tau = \left[\frac{A_1}{\xi} J_2\left(\frac{4K_1}{\sqrt{2\xi}}\right) + \frac{B_1}{\xi} Y_2\left(\frac{4K_1}{\sqrt{2\xi}}\right)\right] \sin(K_1\eta + C_2) \tag{20}$$

where $K_1 = 2k$ and A, B, C and k are arbitrary constants.

x can now be found from (9) giving

$$x - \frac{\eta\tau}{4} = \left[\frac{A}{\xi^{3/2}} J_3\left(\frac{4K_1}{\sqrt{2\xi}}\right) + \frac{B}{\xi^{3/2}} Y_3\left(\frac{4K_1}{\sqrt{2\xi}}\right)\right] \cos(K_1\eta + C_2) \tag{21}$$

where $A = \frac{A_1}{4\sqrt{2}}$, $B = \frac{B_1}{4\sqrt{2}}$, J_3 and Y_3 are third order Bessel Functions of the first and second kind.

The following observations should be made concerning the use of this solution. The boundary and initial conditions must first be transformed to the hodograph plane. Then, the solution must be made to fit the boundary conditions. This, in general, will involve an infinite series of Bessel Functions. Finally, this solution must be transformed back to the physical plane, which, if an infinite series is involved, could only be done numerically. However, this would not be a difficult problem to program on a digital computer.

SOLUTIONS FOR WHICH J VANISHES

There are three principal classes of solutions which are of particular importance. The simplest of these are constant states. Obviously ξ = a constant, η = a constant are solutions of Eqs. 1.

The second class of solutions is sometimes called simple waves. The theory of simple waves is amply discussed in (2). Let us recall that a simple wave is defined as a region in the $x - \tau$ plane which maps onto a single characteristic line in the hodograph plane. A fundamental theorem states that the region adjacent to a region in a constant state (which maps onto a single point) must be a simple wave.

The third class of singular solutions comprises discontinuous solutions which we shall call shock waves because of the similarity to compressible flow theory. Shock waves arise in certain cases where it is impossible to find continuous solutions which satisfy given boundary conditions. If the boundary conditions are physically admissible, additional physical considerations admit a class of discontinuous solu-

tions. As in other theories, these discontinuities arise because some physical effect has been omitted from the mathematical model. For instance, in compressible flow, shocks arise because the viscosity is left out. In this thesis they arise because the axial stress in the tube was left out. We shall show later, however, that the shock waves present a fairly good approximation to the actual physical situation. The discontinuity must obey the physical conservation laws of mass, momentum, energy, and entropy.

We shall now derive the relations which must be satisfied for discontinuous solutions of this problem. In this case, the derivation is based on a technique used in (2); namely, a small element including the discontinuity and moving with it will be considered and the limit of the time derivative of the integral, throughout the column, of the conserved quantity will be taken, as the element is shrunk toward the discontinuity. It is shown that

$$\lim_{(a_1-a_0)\to 0} \frac{d}{d\tau}\int_{a_0}^{a_1} \psi(x,\tau)dx = \psi_1 v_1 - \psi_0 v_0 \tag{22}$$

where the 0 and 1 refer to conditions on either side of the shock, ψ is the quantity conserved, and the v_i are the velocities relative to the shock wave. In this problem

$$v_i = \eta_i - V \tag{23}$$

where V is the velocity with which the shock is moving. The conservation laws take the following form

$$\text{Mass:} \quad \frac{d}{d\tau}\int_{a_0}^{a_1} \xi^2 dx = 0 \tag{24}$$

$$\text{Momentum:} \quad \frac{d}{d\tau}\int_{a_0}^{a_1} \xi^2\eta dx = P_0\xi_0^2 - P_1\xi_1^2 - P_e(\xi_0^2 - \xi_1^2) \tag{25}$$

or; the time rate of change of momentum is equal to the forces acting and the external pressure, P_e, exerts an axial component of force on the shock wave.

$$\text{Energy:} \quad \frac{d}{d\tau}\int_{a_0}^{a_1} \xi^2\left(\frac{\eta^2}{2} + e + \frac{\sigma^2}{2E}\right)dx = P_0\eta_0\xi_0^2 - P_eV(\xi_0^2 - \xi_1^2) - P_1\eta_1\xi_1^2 \tag{26}$$

where e is the internal energy per unit mass and $\frac{\sigma^2}{2E}$ represents the strain energy stored in the tube and the terms on the right represent the power input or the work done per unit time.

Entropy: $$\frac{d}{d\tau}\int_{s_0}^{s_1} \xi^2 S dx \geqslant 0. \tag{27}$$

Now, if relation (22) is applied to (24) through (27), the shock relations become:

Mass: $$\xi_1^2 v_1 = \xi_0^2 v_0 = Q_e \tag{28}$$

where Q_e is the quantity of fluid crossing the shock.

Momentum: $$\xi_1^2 \eta_1 v_1 - \xi_0^2 \eta_0 v_0 = P_0 \xi_0^2 - P_1 \xi_1^2 - P_e(\xi_0^2 - \xi_1^2) \tag{29}$$

which can also be written with the use of (28) and (23) as

$$\xi_1^2(v_1^2 + P_1 - P_e) = \xi_0^2(v_0^2 + P_0 - P_e). \tag{30}$$

Energy: $$\xi_1^2 v_1\left[\frac{\eta_1^2}{2} + e_1 + \frac{\sigma_1^2}{2E}\right] - \xi_0^2 v_0\left[\frac{\eta_0^2}{2} + e_0 + \frac{\sigma_0^2}{2E}\right]$$
$$= P_0 \eta_0 \xi_0^2 - P_1 \eta_1 \xi_1^2 - P_e V(\xi_0^2 - \xi_1^2) \tag{31}$$

Entropy: $$\xi_1^2 v_1 S_1 - \xi_0^2 v_0 S_0 \geqslant 0$$

or with the use of (28) this can be written

$$Q_e(S_1 - S_0) \geqslant 0. \tag{32}$$

A general discussion of these relations is enlightening because the situation is somewhat different from compressible flow. On each side of the discontinuity, we have three quantities, the radius, velocity, and pressure to deal with, as well as the shock velocity V. However, we have equilibrium of the membrane on both sides of the shock. Here we differ from compressible flow where the adiabatic law between pressure and density is not valid across the shock. Thus, we have actually five quantities to deal with. If the state (ξ, η, and P) is given on one side of the shock and one quantity on the other side, then there remain just two unknowns (the remaining quantity in the unknown state and the shock velocity V) to be solved for. These can be found from the first two shock relations—referred to as the mechanical shock conditions. To gain insight into the meaning of the other relations, we must investigate the internal energy function. Assume:

$$e = e(V, T)$$
$$de = \frac{\partial e}{\partial V}\Big)_T dV + \frac{\partial e}{\partial T}\Big)_V dT$$

where V denotes volume. Since we are assuming an incompressible fluid, $dV = 0$. Thus

$$de = C_v dT. \tag{33}$$

Since in Eq. 31, the values of strain energy in the tube can be calculated knowing ξ_i, then the energy equation can be used to calculate the temperature change of the fluid when it crosses the shock; that is,

$$e_1 - e_0 = C_v(T_1 - T_0). \tag{34}$$

The entropy condition can be used to justify which way the jump will occur; that is, there is an indeterminacy in (28) and (29) which in calculations shows up when V is solved for in (29). There is a choice of sign to be made which does not affect (28) at all. This choice must be made to guarantee no decrease in entropy, though it is usually obvious in a given problem which way the jump occurs.

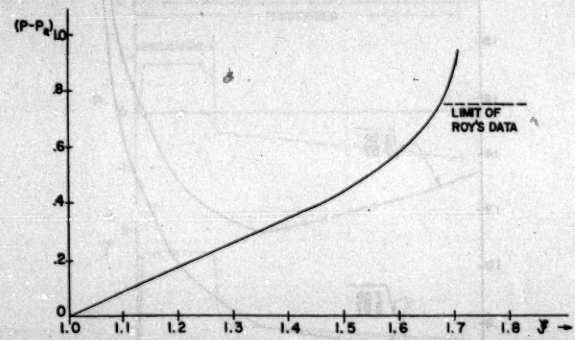

FIG. 1. Gage pressure *versus* radius for the aorta of a twenty-six year old male.

WAVE PROPAGATION IN THE AORTA

We now apply the general theory to the propagation of a pulse wave in the aorta. Since peak systolic pressures are within the most nonlinear portions of the stress-strain curve, it is necessary to take full account of these nonlinearities. Consequently, data were taken from Roy (9) for a twenty-six year old male. The load deflection diagram of Roy was translated into a pressure *vs.* radius curve which is shown in Fig. 1. Sample calculations are given in Appendix A. To make use of this general relationship we write Eqs. 12 and 3 in the following form,

$$\frac{d\eta}{d\xi} = \sqrt{\frac{2}{\xi}\frac{dP}{d\xi}}; \quad \frac{dx}{d\tau} = \eta - \sqrt{\frac{\xi}{2}\frac{dP}{d\xi}} \quad \text{along } \mu = \text{constant}$$

$$\frac{d\eta}{d\xi} = -\sqrt{\frac{2}{\xi}\frac{dP}{d\xi}}; \quad \frac{dx}{d\tau} = \eta + \sqrt{\frac{\xi}{2}\frac{dP}{d\xi}} \quad \text{along } \sigma = \text{constant}. \tag{35}$$

The curves for $\sqrt{\frac{2}{\xi}\frac{dP}{d\xi}}$ and $\sqrt{\frac{\xi}{2}\frac{dP}{d\xi}}$ were computed from Fig. 1 and are shown in Fig. 2.

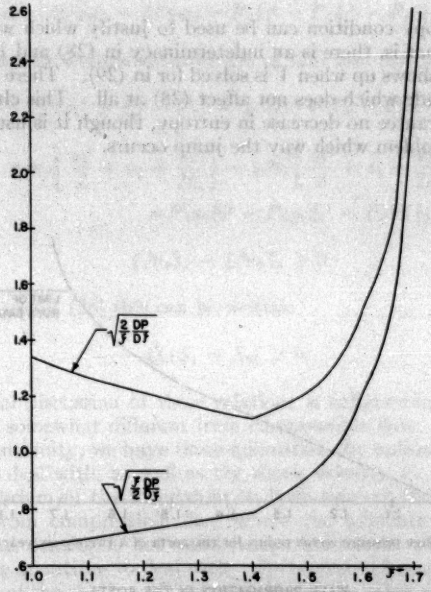

FIG. 2. Functions used to calculate the characteristic slopes.

Now it is worthwhile examining the above equations for the wave velocities. If $\eta > \sqrt{\frac{\xi}{2}\frac{dP}{d\xi}}$ both waves travel forward at different velocities. This condition was found to exist during the whole time of the pulse and throughout most of the aorta. In particular, during the systole (the portion of the pulse during which blood is forced into the artery) at the entrance to the aorta where the velocity is greatest, both waves travel forward. To work the problem then, both ξ and η (or P and η since ξ can be found from P by Fig. 1) must be prescribed during the systole.

Since there were no data available on the velocity during the systole, the velocity was taken as an almost square pulse, as shown in Fig. 3. The magnitude of the pulse was calculated to give a volume of flow of 65 cm.³ of blood during the pulse (1).

During the diastole (the portion of the pulse during which the heart relaxes), we know that the velocity at $x = 0$ will be zero, since we assume the semilunar valve to be closed. We see now that one wave velocity will be negative and one positive, and one datum prescribed on the line $x = 0$ will be sufficient to determine the solution, provided at least one of the μ characteristics originating at $x = 0$ during the

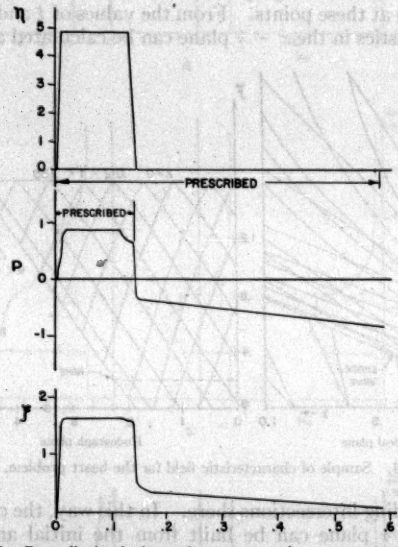

FIG. 3. Prescribed velocity and pressure at the entrance to the aorta.

systole will curve around and intersect $x = 0$ at a later time. Since this happens, the pressure (and/or radius) is determined throughout the whole region until the next pulse. This solution was carried out graphically and a sample of the characteristic fields obtained is shown in Fig. 4.

When the next pulse starts, we have the familiar problem of the characteristics forming an envelope. Thus, we have to treat this as a shock discontinuity. The shock wave encountered in this problem is somewhat curved (in the $x - \tau$ plane) which necessitated the rather time consuming process of plotting it. Essentially, one must locate all the points of two solutions which will satisfy the shock conditions and

lie on a line with slope equal to V, the velocity of the shock. This was done by trial and error and when completed, found to be nearly straight for $x > 0.3$. X was made dimensionless in such a way that $x = 1$ represents the end of an aorta about eight inches long.

The procedure for solution was as follows. The points representing the initial conditions, that is, corresponding values of ξ and η at various values of time at $x = 0$, were plotted in the hodograph plane. Through these points, characteristic curves are drawn with the aid of Eqs. 35 and Fig. 3. These points are also plotted in the $x - \tau$ plane and the characteristic lines started. Now, the intersections of the two families of characteristics in the hodograph determine a new set of points and the values of ξ and η at these points. From the values of ξ and η, the slopes of the characteristics in the $x - \tau$ plane can be calculated and corrected

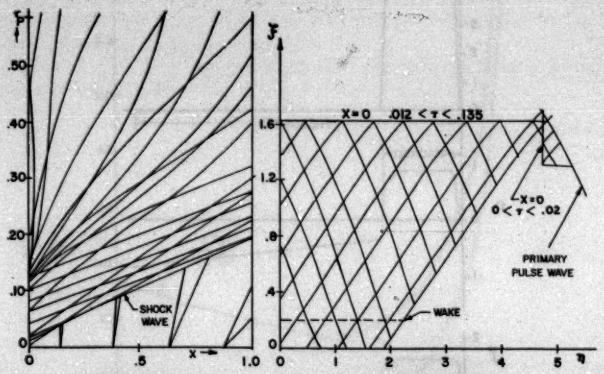

FIG. 4. Sample of characteristic field for the heart problem.

at the corresponding intersections there. In this way, the characteristic field in the $x - \tau$ plane can be built from the initial and boundary values.

At the end of the systole, the μ characteristics near $x = 0$ are found to curve back intersecting $x = 0$ a second time. Knowing that here $\eta = 0$ this point is also known in the hodograph plane so a new σ characteristic is determined from this point. In this way, the solution is completely determined up until the next pulse starts.

One other point bears mentioning, that is, it is possible, with a change of variable, to make a correspondence of the slopes of the characteristics in the $x - \tau$ plane to those of the characteristics in the $\xi - \eta$ plane. However, this introduces the necessity of making an additional calculation to get the values of the functions after the solution is completed. With the method outlined above, it is only necessary to locate the

FIG. 5. Plot of pressure, velocity and radius at $x = 0.25$.

FIG. 6. Plot of pressure, velocity and radius at $x = 0.5$.

FIG. 7. Plot of pressure, velocity and radius at $x = 0.75$.

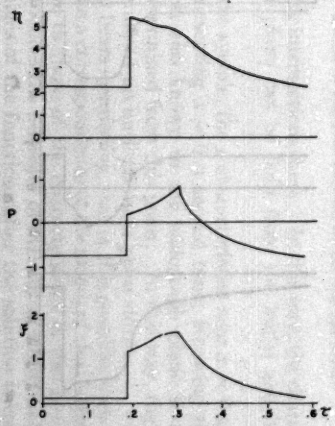

FIG. 8. Plot of pressure, velocity and radius at $x = 1.00$.

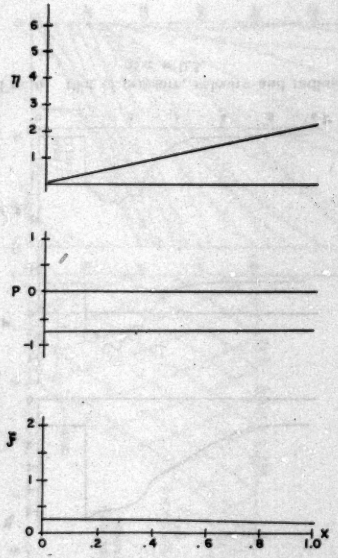

FIG. 9. Plot of pressure, velocity and radius at $\tau = 0$.

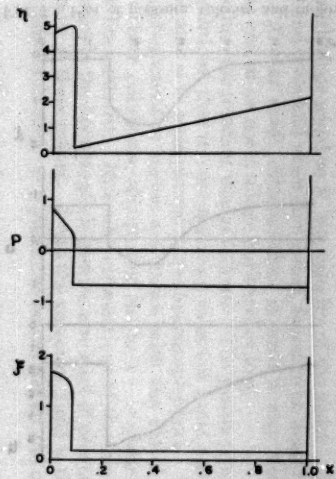

FIG. 10. Plot of pressure, velocity and radius at $\tau = 0.02$.

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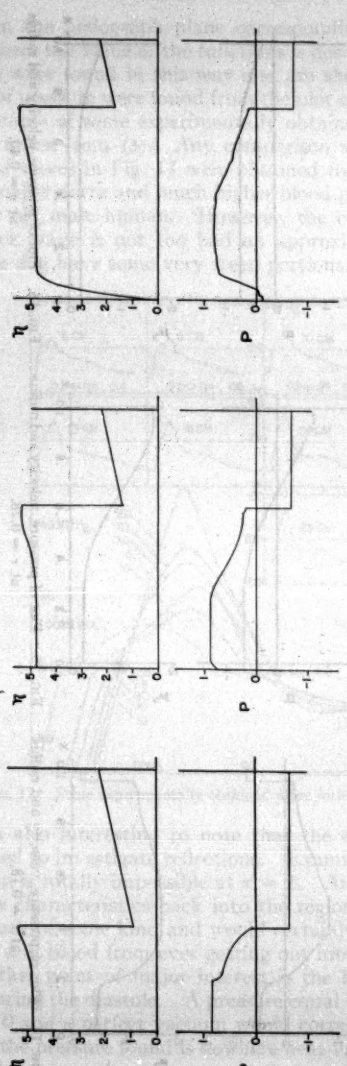

FIG. 11. Plot of pressure, velocity and radius at $\tau = 0.08$.

FIG. 12. Plot of pressure, velocity and radius at $\tau = 0.12$.

FIG. 13. Plot of pressure, velocity and radius at $\tau = 0.14$.

FIG. 14. Plot of pressure, velocity and radius at $\tau = 0.18$.

FIG. 15. Plot of pressure, velocity and radius at $\tau = 0.28$.

FIG. 16. Plot of pressure, velocity and radius at $\tau = 0.40$.

points in the hodograph plane corresponding to points in the $x - \tau$ plane where the value of the functions is desired and pick off the values. ξ and η were found in this way and are shown in Figs. 5 through 16. Values of pressure were found from the plot of ξ and the use of Fig. 1.

A sample of some experimentally obtained wave forms is shown in Fig. 17 taken from (3). Any comparison should be only qualitative, since the waves in Fig. 17 were obtained from a small dog who had a much smaller aorta and much higher blood pressure than has a twenty-six year old male human. However, the comparison does show that the shock wave is not too bad an approximation. The wave forms from the dog have some very steep portions.

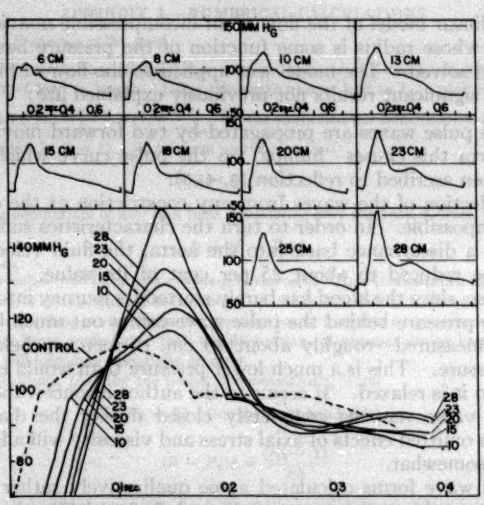

FIG. 17. Some experimentally obtained wave forms from a dog.

It is also interesting to note that the experiments, Fig. 17, were performed to investigate reflections. Examination of Fig. 4 shows that reflection is totally impossible at $x = 1$. Any constriction which could turn the characteristics back into the region $x < 1$ would have to set up a shock of some kind and would certainly be a major factor in preventing any blood from ever getting out into the body.

Another point of major interest is the low pressures found in the aorta during the diastole. A pressure equal to atmospheric corresponds to $P = 0$ and a perfect vacuum would correspond to $P = -5.02$. So though the pressure found is nowhere near vacuum, it is much less than is usually measured.

Admittedly, the omission of viscosity and axial stress will cause this pressure to come out somewhat below normal. Physically, however, it would seem natural to expect low pressure behind such a comparatively tremendous pulse. On the other hand, this pressure would not be so low if we had not prescribed a zero velocity at $x = 0$ during the diastole. It would seem logical that the semilunar valve may not remain closed but may allow a trickle of blood to be sucked into the aorta by the pulse. There is some unpublished experimental evidence to the effect that the semilunar valve does reopen quickly once or twice after it first closes.

CONCLUSIONS

A nonlinear model of the flow of an incompressible nonviscous fluid in a tube whose radius is some function of the pressure has been discussed and solved. The model was applied to the flow of blood in the aorta and significant results not previously explained are:

1. The pulse waves are propagated by two forward moving waves. In the aorta this causes "humps" on the pulse curve which had previously been ascribed to reflection (3, 4, 5).
2. Reflection of the waves from any constriction at the end of the aorta is impossible. In order to turn the characteristics sufficiently to propagate a disturbance back into the aorta, the fluid velocity would have to be reduced to about 25 per cent of its value. This hardly makes sense, since the blood has barely started its journey into the body.
3. The pressure behind the pulse wave comes out much lower than has been measured—roughly about 10 cm. of mercury below atmospheric pressure. This is a much lower pressure than would exist in the heart when it is relaxed. It seems to the author inconceivable that the semilunar valve remains completely closed during the diastole. Of course, the omitted effects of axial stress and viscosity will alleviate this condition somewhat.
4. The wave forms calculated agree qualitatively rather well with published experimental curves (see Figs. 3, 5, 8 and 17).

Acknowledgments

The author wishes to thank his graduate committee headed by Dr. A. C. Eringen for help with the research and Dr. N. O. Myklestad for reviewing the manuscript. The paper is dedicated to "Uncle Hubie," a lifelong friend who died recently of heart disease.

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APPENDIX A.—NUMERICAL CALCULATIONS

I. Mechanical Properties of the Aorta

The curve given in Roy (9) for a strip of aorta taken from a twenty-six year old male is actually a plot of load *vs.* length. Using the length at zero load as L_0, a plot was made from this of the load Q *vs.* $\frac{L - L_0}{L_0}$. Now $\frac{L - L_0}{L_0}$ is the strain and our dimensionless form of strain in terms of radius is $(\xi - 1)$. So we have now the curve

$$Q = Q(\xi - 1). \tag{A-1}$$

From the equilibrium of any thin tube, considering only the hoop stresses, we have

$$Q = (P - P_a)r \tag{A-2}$$

if Q is the circumferential force on an element of the tube. Here it is assumed that P_a (atmospheric pressure) is acting external to the aorta. Actually this will fluctuate somewhat because the aorta is in the chest cavity where the pressure has to fluctuate to cause breathing. However, the heart beats much faster than the fluctuations of pressure during breathing so it is felt that this will introduce essentially no error.

If we now divide (A-2) by the proper dimensionalizing factor

$$\rho\alpha R_0 = Eh \quad \text{(for the right hand side)}$$

we have

$$(P - P_a)\xi = \frac{Q(\xi - 1)}{Eh}. \tag{A-3}$$

Now Roy does not give any data for h, the thickness of the aorta, which he tested. However, he does state that the strips were 1 cm. wide. We note that

$$E = \frac{\sigma}{\epsilon} = \frac{Q/h \cdot 1}{\xi - 1} \tag{A-4}$$

so that the dimension factor Eh needed in (A-3) is just

$$Eh = \frac{Q}{\xi - 1} \tag{A-5}$$

and is equal to the slope of the curve we have just described. This slope was found to be 156 gm/cm for the curve in the transverse direction. Thus the curve $(P - P_a) = (P - P_a)(\xi)$ was calculated and plotted from

$$(P - P_a) = \frac{Q(\xi - 1)}{156\xi} \tag{A-6}$$

(see Fig. 1).

An attempt was made to fit the curve $Q(\xi - 1)$ with a simple analytic expression. An excellent fit was obtained up to $\xi - 1 = 0.52$ by using

$$Q = Eh(\xi - 1) + \beta(\xi - 1)^3. \tag{A-7}$$

However, this curve diverged rapidly for $\xi - 1 > 0.52$ and this is the region of peak systolic pressures, so this was not pursued further.

Next, the functions $\sqrt{\frac{\xi}{2}\frac{dP}{d\xi}}$ and $\sqrt{\frac{2}{\xi}\frac{dP}{d\xi}}$ were calculated at as many points as were necessary to plot the curves shown in Fig. 2. Here, the slopes $\frac{dp}{d\xi}$ were measured graphically from the curve in Fig. 1. Several measurements were repeated at each point to reduce graphical error.

II. Boundary Conditions

A curve of pressure *vs.* time for both the heart and aorta was found in Best and Taylor (1). The curve for the pressure during the systole was taken as accurately as it could be from a rather rough plot. From these data and the curve in Fig. 1 the radius at $x = 0$ was calculated and is shown in Fig. 3.

No data were available on the velocity, except that the heart usually pumps 60–70 cc. of blood per pulse. Therefore, a nearly square-wave velocity distribution was assumed and the magnitude η^* calculated such that

$$\int_0^{t_*} \pi r_0^2 \eta^* dt = 65 \text{ cm}^3. \tag{A-8}$$

This integration was carried out numerically using twenty equal time increments t_i and making the following approximation,

$$\pi R_0^2 \eta^* \sum_{i=0}^{20} \xi_i^2 \Delta t_i = 65 \text{ cm}^3 \tag{A-9}$$

or

$$\eta = \frac{\eta^*}{\sqrt{\alpha}} = \frac{65}{\pi R_0^2 \Delta t_i \sqrt{\frac{Eh}{\rho R_0}} \sum_i \xi_i^2}$$

with $R_0 = 0.795$ cm. (computed from circumference given by Roy)

$\Delta t_i = 0.01$ sec
$Eh = 156$ gm/cm
$\rho = 1.05$ (for blood given by Best and Taylor)
$\eta = 4.79$ (was taken as 4.8 to make up for slight lack of squareness).

During diastole, the only boundary condition we need is $\eta = 0$ at $x = 0$, and this assumes that the semilunar valve remains closed.

The time was made dimensionless by setting

$$\tau = \frac{t\sqrt{\alpha}}{L_0}. \tag{A-10}$$

L_0 was taken to be 8 in. = 20.3 cm. as being approximately the length of the aorta before it branches.

This gives $\tau = 0.685\, t$
Thus, one second gives $\tau = 0.685$
Ejection time 0.2 seconds gives $\tau_e = 0.137$
Pulse time $0.685 \frac{60 \text{ sec}}{70 \text{ pulses}}$ gives $\tau_p = 0.857$/pulse.

SPUTNIK AS A TOOL FOR SECURING GEODETIC INFORMATION

BY

LOUIS GOLD[1]

ABSTRACT

A simplified picture is employed for utilizing Sputnik tracking data (perigee, apogee, period, maximum and minimum orbital velocity) in the determination of the Earth's mass and radius. For this purpose, basic characteristics of the motion are conveniently re-expressed in terms of the perigee and apogee parameters. Means for deducing the launching conditions are also provided and from reported perigee, apogee data for Sputnik I it is inferred that horizontal launching took place at a firing velocity slightly more than the lower bound value.

1. INTRODUCTION

In an earlier work (1)[2] there was indicated the manner in which satellite orbits are prescribed in terms of the launching angle and velocity. The purpose of the present report is to describe how Sputnik tracking data which include perigee and apogee, period and orbital velocity may be used to deduce important geodetic facts such as the mass and mean radius of the Earth without any knowledge of the launching conditions. Means for delineating the launching conditions themselves will also be pointed out.

Fine geodetic details involving departures of the Earth's shape from true sphericity can only be gotten from perturbation methods (2). No effort will be made in this direction, nor will consideration be given to using all available Sputnik data for analysis of the distribution of land masses, etc.

2. CHARACTERISTICS OF SATELLITE ORBITS EXPRESSED IN TERMS OF PERIGEE AND APOGEE

Let the elliptic orbit be given in the polar form

$$r = \frac{(d + l)e}{1 - e\cos\theta} \tag{1}$$

where r, θ are the coordinates referred to the Earth's center and e is the eccentricity; d is the directrix distance and l is the true perigee. It can readily be demonstrated that the semi-major and minor axes are

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

related to perigee l and apogee L distances as follows:

$$\begin{aligned} a &= \tfrac{1}{2}(l + L) \\ b &= \sqrt{Ll}. \end{aligned} \tag{2}$$

Also it can be shown that

$$d = l\left(\frac{L + l}{L - l}\right) \tag{3}$$

$$e = \frac{L - l}{L + l} \tag{4}$$

whereby (1) itself may be written in the form

$$r = \frac{2Ll}{(L + l) - (L - l)\cos\theta}. \tag{5}$$

In turn, the perigee and apogee may be related to the launching angle φ and velocity v_0 via

$$\begin{aligned} a &= \frac{\frac{1}{2}r_0}{1 - v_0^2/v^{*2}}, \quad b = \frac{r_0 \dfrac{v_0}{v^*}\sin\varphi}{(1 - v_0^2/v^{*2})^{1/2}} \\ 1 - e^2 &= 4\frac{v_0^2}{v^{*2}}\left(1 - \frac{v_0^2}{v^{*2}}\right)\sin^2\varphi \end{aligned} \tag{6}$$

where r_0 is the Earth's radius and v^* is the escape velocity (1). Making use of (2) and (3) in combination with (6) leads to

$$L, l = \tfrac{1}{2}\frac{r_0}{1 - v_0^2/v^{*2}}\{1 \pm \sqrt{1 - 4v_0^2/v^{*2}(1 - v_0^2/v^{*2})\sin^2\varphi}\} \tag{7}$$

where clearly the positive sign pertains to L and negative sign for l.

Another useful relation that will be needed is the period T.

$$T = \pi\frac{r_0}{v^*}\left(1 - \frac{v_0^2}{v^{*2}}\right)^{-3/2}. \tag{8}$$

Next the orbital velocity must be established.

3. NATURE OF THE ORBITAL VELOCITY

The basic relation has

$$\frac{ds}{dt} = v = \frac{ds}{d\theta}\cdot\frac{d\theta}{dt} \tag{9}$$

where $ds/d\theta$ comes from the length of arc expression

$$s = \int \left[r^2 + \left(\frac{dr}{d\theta} \right)^2 \right]^{1/2} d\theta. \tag{10}$$

Thus

$$\frac{ds}{d\theta} = r \left[1 + \left(\frac{e \sin \theta}{1 - e \cos \theta} \right)^2 \right]^{1/2}$$

$$= r^2 \left(\frac{L + l}{2L} \right) \left[1 + \left(\frac{L - l}{L + l} \right)^2 - 2 \frac{L - l}{L + l} \cos \theta \right]^{1/2}. \tag{11}$$

Then from Kepler's law

$$r^2 \frac{d\theta}{dt} = \frac{2\pi ab}{T} \tag{12}$$

whence finally for the orbital velocity there obtains

$$v = \frac{\pi}{T} \cdot \frac{(L + l)^2}{\sqrt{2lL}} \left[1 + \left(\frac{L - l}{L + l} \right)^2 - 2 \frac{L - l}{L + l} \cos \theta \right]^{1/2}. \tag{13}$$

The periodic behavior for v contains extremals at $\theta = n\pi$. Indeed

$$v_{max} = \frac{\pi}{T} (L + l) \sqrt{\frac{L}{l}} \tag{14a}$$

$$v_{min} = \frac{\pi}{T} (L + l) \sqrt{\frac{l}{L}} \tag{14b}$$

with the interesting outcome

$$\frac{v_{max}}{v_{min}} = \frac{L}{l}. \tag{15}$$

As a check observe that $L/l \to 1$ properly reduces (13) to the anticipated circular velocity $v = 2\pi R/T$.

4. GEODETIC CALCULATIONS FROM SPUTNIK DATA

Suppose the maximum and minimum radar heights are L' and l', respectively, with

$$\begin{aligned} L &= L' + r_0 \\ l &= l' + r_0. \end{aligned} \tag{16}$$

Then from the orbital velocity data (deduced from Doppler shift of radio frequency), Eq. 15 very readily permits determination of the Earth's radius.

The mass of the Earth M can be found by making use of the energy conservation relation

$$\tfrac{1}{2}mv_{\max}^2 + \frac{\gamma Mm}{L} = \tfrac{1}{2}mv_{\min}^2 + \frac{\gamma Mm}{l} \tag{17}$$

where γ is the gravitational constant. Incorporating Eq. 14 yields

$$M = \left[\frac{\pi}{T}(L+l)^2\right]\cdot\frac{L+l}{2\gamma} = \left(\frac{\pi}{T}\right)^2\cdot\frac{1}{2\gamma}(L+l)^3. \tag{18}$$

Hence from the observed period and Eq. 16 with the known value of γ, the Earth's mass is calculable.

5. DISCUSSION

The simple analysis presented affords an insight as to how Sputnik tracking data may be utilized to gain fundamental knowledge of the Earth's properties. Of course refinements in the theory of orbital motion will allow maximum use of the vast collection of data in arriving at a detailed picture of our planet. Attention here has been directed to perceiving the gross features only in terms of an effective mass and radius of the Earth.

With r_0 prescribed, the launching parameters v_0 and φ may be found. After some manipulation of the foregoing analysis it follows that

$$v_0 = v^*\sqrt{\frac{L'+l'+r_0}{L'+l'+2r_0}} \tag{19}$$

$$\varphi = \sin^{-1}\sqrt{\frac{(L'+r_0)(l'+r_0)}{r_0(L'+l'+r_0)}}. \tag{20}$$

The observed perigee and apogee distances for Sputnik I soon after launching were

$$L' = 560 \text{ miles}, \quad l' = 140 \text{ miles}$$

which combined with $r_0 = 3960$ miles yield

$$\frac{v_0}{v^*} = \sqrt{\frac{4660}{8620}} = 0.74$$

and

$$= \sin^{-1}\sqrt{\frac{(4520)(4100)}{(3960)(4660)}} = \sin^{-1} 1$$

or $\varphi = \frac{\pi}{2}$. Thus it appears that Sputnik I was put into its orbit by a

horizontal firing at slightly in excess of the minimal satellite velocity $v_0/v^* = 0.706$.

If launching heights comparable to or greater than the Earth's radius are entailed, the foregoing results must be altered appropriately (1). It is doubtful that Sputnik I was sent into its orbit at a height greater than a few hundred miles.[3]

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[3] Note added after review and acceptance of manuscript:
An article "Soviet Moon" (3) revealed the following information about Sputnik I:

1. Launching angle—90°
2. Launching height—200 km. (124 miles)
3. Orbital velocity—8 km/sec (5 mi/sec); $v_0/v^* = 0.74$.

Nylon Hands Help Nurses to Lift.—A new type of hoist with nylon "hands" that can lift patients has been designed in Britain to relieve strain upon nurses and to simplify hospital porterage.

The hydraulic multi-purpose hoist is actually part of a wheeled stretcher designed to go over and under the bed. The invention is the product of the combined ideas of the medical staff of a Norfolk (England) hospital and technicians of a Norwich (England) engineering firm. Dr. J. V. Morris, medical superintendent of the hospital at Little Plumstead, realized the daily risks of injuries to nurses in their difficult task of moving heavily built patients. He is largely responsible for the invention. The apparatus has been named the "Little Plumstead Patient Lifter" and is being made by Mann Egerton & Co., Norwich, England.

Moving a heavy, immobile patient from his bed to another part of the hospital used to be a task for two or three nurses. Now all that is needed is for a nurse to roll the nylon hands under a patient, touch a handle controlling the hydraulic power and the patient is gently hoisted from bed.

There are two heads for use with this lifter, one horizontal and the other vertical. When the former is used, the patient can be lifted from bed and transferred via the lifter direct from the ward to the operating theatre.

The vertical head has been designed for moving patients in a sitting position by means of a harness. It calls for no cooperation or effort from the patient and makes it a simple matter for the nurse to lift and then lower her patient into a bath or on to a wheel-chair.

New Plastic Material.—A new plastic material that combines desirable properties of rubber, nylon and steel was described recently at an ASME meeting in Detroit. Known commercially as "Vulkollan" the substance is said to combine "hardness, strength, rigidity, resilience, elasticity, load carrying, abrasion resistance, oil resistance and oxygen or ozone resistance as no other material has ever done before."

According to Kenneth A. Pigott of the Mobay Chemical Company, New Martinsville, W. Va., the substance will snap back to its original size after being stretched to five or six times its original length.

Its peculiar combination of qualities make the plastic valuable in such applications as handles for air hammers like those used to tear up pavement. "The handles are sufficiently rigid so that no wobble is present and at the same time vibration transmitted to the worker's arms is drastically reduced. Where these hammers have been used worker fatigue and absenteeism have improved," according to Mr. Pigott.

The same substance can be used to make solid tires for industrial trucks that carry a heavier load, reduce power consumption of the truck by as much as 50 per cent and last longer than conventional rubber tires. In actual use, "Volkollan" will resist abrasion ten times better than good natural rubber.

The new material has found wide usage in European markets for several years but has been introduced in this country only recently. The new plastic has been used in Germany to make motorcycle sprocket gears. The gear itself is thin and flexible, but the gear teeth are rigid enough to drive the motorcycle. Such gears have lasted for over 60,000 miles of operation with no appreciable sign of wear.

ON FLIGHT TRAJECTORIES IN THE NEIGHBORHOOD OF A KNOWN TRAJECTORY

BY

R. M. ROSENBERG [1]

INTRODUCTION

The function of a pilotless aircraft is usually of such a nature that it must occupy a given position in space and time with reference to a fixed point in the space-time manifold; for instance the launch time and position, or the time and position of the initiation of a maneuver furnish such a reference point. The result of having the aircraft occupy a position (x_i, y_i, z_i) at the time t_i is accomplished by programming the flight in such a manner that its trajectory $x(t)$, $y(t)$, $z(t)$ passes through the point (x_i, y_i, z_i) when $t = t_i$. Clearly, one can rely on this coincidence to occur only if one can rely on the predictions of the trajectory which results from a given program of the controls. This prediction consists in integrating the equations of motion of the aircraft.

The equations of motion are strongly nonlinear ordinary differential equations whose coefficients are not only complicated functions of several dependent variables, but of time as well. Therefore, the integrations are not simple; they require considerable experience, patience and accuracy from the computer. Moreover, the equations of motion are six in number for a rigid aircraft—more than six for an elastic one—and this large number of equations makes the task of their integration forbidding even when modern electronic digital computers are used. In routine computations, the number of equations is usually reduced to two. Three of them are eliminated by constraining the flight (of a symmetric aircraft) to a vertical plane, and an additional equation is shed by replacing the missile by a point-mass. (This latter step is equivalent to setting the pitching moment of inertia equal to zero!)

Assuming for the present that the remaining two equations represent properly the flight dynamics, their integration yields the correct flight trajectory only if all parameters of the problem, all functional relations implicit in the coefficients of the equations, and all initial conditions are known with precision. Knowledge of this sort implies that the temperature variation with altitude on the day of the flight be known at the time when the computations are carried out, that one have knowledge of the velocity, altitude and weight of the aircraft at the instant of maneuver initiation, that each individual engine perform exactly as specified, that the program of the controls take place precisely as planned, etc., etc. But these factors depend on vagrancies of

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the day and climate, on manufacturing idiosyncracies and imperfections, and on the degree of validity of many assumptions; at best, they can be expected to be known within a statistical distribution only.

This lack of precise knowledge regarding the quantities which determine the trajectory leads to the necessity of computing trajectories for conditions neighboring on the nominal ones, and it is this problem which is our subject.

THE NOMINAL TRAJECTORY

Consider an airplane on a flight path neighboring on the nominal one. This is shown in Fig. 1. The x_0, y_0-axes are the unperturbed wind

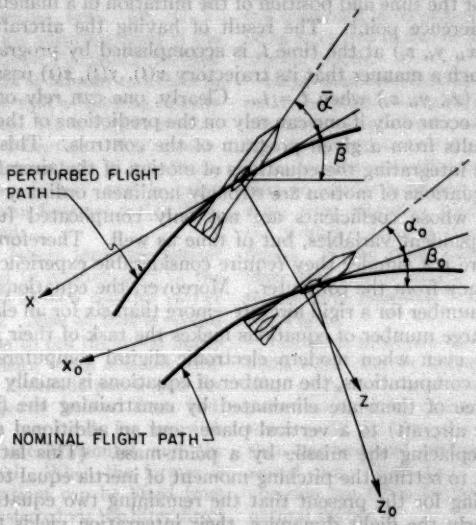

FIG. 1.

axes. The x_0-axis is positive in the direction opposing drag, the z_0-axis is positive in the direction opposing up-lift. The perturbed wind axes are the x, z-axes. The inclination of the unperturbed (nominal) flight path to the horizontal is β_0, that of the perturbed (neighboring) flight path to the horizontal is $\bar{\beta}$. The perturbed angle of attack $\bar{\alpha}$ is the angle between the x-axis and the body axis along which the perturbed thrust $\bar{T}$ is taken to act. All angles are positive in a nose-up direction.

The derivation of the equations of motion for this system is easily accessible and need not be reproduced here. For the unperturbed

flight, for instance, the equations are

$$\frac{dV_0}{dt} = -g \sin \beta_0 - \frac{D_0}{m_0} + \frac{T_0}{m_0} \cos \alpha_0 \tag{1}$$

$$-V_0 \frac{d\beta_0}{dt} = g \cos \beta_0 - \frac{L_0}{m_0} - \frac{T_0}{m_0} \sin \alpha_0 \tag{2}$$

$$I_0 \frac{d^2\theta_0}{dt^2} = M_0 \tag{3}$$

where

V_0 = tangential velocity,
g = acceleration of gravity,
m_0 = airplane mass,
I_0 = pitching mass moment of inertia,
D_0 = drag,
L_0 = lift,
T_0 = thrust,
M_0 = pitching moment,
$\theta_0 = \alpha_0 + \beta_0$

and the x_0, z_0-system is fixed in the airplane with the origin at the center of gravity. The first of these equations sums the forces along the flight path, the second those normal to the flight path, and the third the moments.

Equations 1, 2 and 3 are all nonlinear. For instance, the quantities L_0, D_0, M_0 and T_0 are all nonlinear functions of the translational velocity V_0, of the air density ρ which depends in a nonlinear fashion on the altitude, and of such variables as α_0, θ_0 and others. Furthermore, if an acceleration or angle of attack program is prescribed, or if the throttle position is a predetermined function of time, many quantities in the equations may become explicit functions of time as well.

Although the equations are so involved that their "closed form" integration may be confidently regarded as hopeless, they can, of course, be integrated by numerical methods for specified initial conditions; this becomes evident when one writes them in the form

$$\left.\begin{aligned} \frac{dV_0}{dt} &= f_1(V_0(t), \beta_0(t), \theta_0(t)) \\ \frac{d\beta_0}{dt} &= f_2(V_0(t), \beta_0(t), \theta_0(t)) \\ \frac{d^2\theta_0}{dt^2} &= f_3(V_0(t), \beta_0(t), \theta_0(t)) \end{aligned}\right\}. \tag{4}$$

If trajectories are computed for a point-mass missile for which $I_0 = 0$, Eq. 3 separates from the system, and obvious simplifications in Eqs. 4 result. We shall assume in everything which follows that this

was done, that is, that the pitching degree of freedom was ignored in computing the nominal trajectory.

Suppose, then, that the system of equations in question consists of Eqs. 1 and 2 only and that these have been integrated for the initial values $V_0(0)$ and $\beta_0(0)$. In this event, V_0 and β_0 are known functions of time, and every quantity of interest regarding the trajectory is known as well, or else can be computed. In particular, the angle of attack α_0, the altitude h_0, the range (horizontal distance from a reference point) r_0, and the Mach number $\frac{V_0}{a_0}$ are all fully defined functions of time. This trajectory, about which everything worth knowing is known, will be called the "nominal trajectory" and the value of every function or parameter pertaining to it will be called the "nominal value" of that function of parameter. All nominal parameters and variables have a zero subscript.

THE VARIATIONAL EQUATIONS

Our interest centers on the computation of trajectories which lie in the neighborhood of a nominal one, and we shall call these "neighboring trajectories." The value of every function, variable and parameter of a neighboring trajectory will be called the "neighboring value," and neighboring quantities are denoted by a superscript bar. In general, it will be assumed that neighboring quantities lie close enough to the nominal ones so that linear theory is adequate to predict the effects on the trajectory.

The methods applied in this paper are well known; they are the so-called perturbation methods. There exists an enormous volume of literature dealing with perturbation theory in general, and a number of contributions deal with the application of these methods to problems of flight. However, most authors who have worked in trajectory perturbations have considered special problems. For instance, Jones (1)[1] presents the application of the perturbation calculus to the flight of airplanes whose performance is independent of compressibility effects, of temperature and of pressure variations, and whose mass is constant, and his interest centers mainly on questions of dynamic stability. Moulton (2) and Bliss (3) are primarily concerned with mathematical methods in ballistics, but they restrict their treatment to missiles having zero life and thrust.

Kooy and Uytenbogaart (4) use perturbation methods to account for slight effects in extra-terrestrial missile dynamics, and for earth rotation in the case of terrestrial trajectories. Drenick (5) presents a rather basic approach to the application of perturbation calculus in the two-dimensional problem of missile ballistics, but it is evident that his main concern is the "ballistic missile," and nearly two-thirds of his

[1] The boldface numbers in parentheses refer to the references appended to this paper.

paper deal with the "cut-off criterion" and questions of guidance after "cut-off."

It is our purpose to present methods and results in two-dimensional missile ballistics which have considerable generality, and we shall derive formulas for computing trajectories neighboring on nominal ones regardless of the cause, or combination of causes, from which the perturbations arise.

A trajectory which lies in the neighborhood of, but does not coincide with, a nominal trajectory differs from it because it satisfies different differential equations, or different initial conditions, or both. We wish to describe a method applicable to the computation of a neighboring trajectory with great freedom as regards the cause which makes it different from the nominal one; therefore, we do not specify here what the disturbing factor is. However, it is appropriate to point out that there are two types of causes which may give rise to the perturbation. In one, it is possible to demonstrate at the outset that the new trajectory will indeed lie in the neighborhood of the nominal one. Included in this type of disturbances are sufficiently small changes in the parameters, coefficients or initial conditions. In these cases, a well-known theorem, applicable to our equations, states that the solutions of the differential equations are continuous functions of the quantities which are subjected to small changes. Therefore, these small changes can produce at most small perturbations in the trajectories. The other type involves changes in parameters whose role in the equations is such that the integrals are not sensitive to their magnitude. Where this is the case, it may be possible to make large changes in a parameter without affecting the trajectory significantly. However, the introduction of a large change in a parameter together with an application of the methods of this paper involves the *a priori* assumption that a significant change from its nominal value in a parameter produces but a small change in the trajectory. In these cases, one must then show *a posteriori* that the initial assumption is born out by the results.

The methods of this paper are well established. They consist in deriving and integrating the so-called equations of the variation. This derivation proceeds by replacing every quantity in Eqs. 1, 2 and 3 which has a zero subscript with one that has a bar. Every barred quantity $\bar{q}$ is of the form

$$\bar{q} = q_0 + q \tag{5}$$

where $|q| \ll |q_0|$ for all t. (Where this condition is violated, it may still be possible, as shown later, to apply the methods to be described.) The resulting equations are expanded in powers of small quantities and only first order terms are retained. Finally, the "nominal" Eqs. 1, 2 and 3 are each subtracted from the corresponding neighboring one.

When this is done the perturbation equations become

$$\frac{dV}{dt} = -g\beta\cos\beta_0 + \frac{D_0}{m_0}\frac{m}{m_0} - \frac{D}{m_0} - \frac{T_0}{m_0}\left(\alpha\sin\alpha_0 + \frac{m}{m_0}\cos\alpha_0\right) + \frac{T}{m_0}\cos\alpha_0 \quad (6)$$

and

$$-V\frac{d\beta_0}{dt} - V_0\frac{d\beta}{dt} = -g\beta\sin\beta_0 + \frac{L_0}{m_0}\frac{m}{m_0} - \frac{L}{m_0} - \frac{T_0}{m_0}\left(\alpha\cos\alpha_0 - \frac{m}{m_0}\sin\alpha_0\right) - \frac{T}{m_0}\sin\alpha_0 \quad (7)$$

and

$$\frac{d^2\theta}{dt^2} = \frac{M}{I} - \frac{d^2\theta_0}{dt^2}. \quad (8)$$

While we still define $I = I_0 + I$, the condition $0 < I \ll I_0$ has been violated in Eq. 8. This is an instance where the supposition that the magnitude of I has a small effect on the trajectory must be reviewed in the light of the results.

The variational equations which have been obtained are written with respect to the perturbed x, z-system of Fig. 1. In other words, Eq. 6 sums the perturbation forces parallel to x, and Eq. 7 those parallel to z. Since we wish to refer the perturbations to the nominal trajectory, we must rewrite the perturbation equations for the nominal x_0, z_0-system. These new perturbation equations are

$$a_x = \left(\frac{m}{m_0} - \frac{D}{D_0}\right)\frac{D_0}{m_0} - \beta\frac{L_0}{m_0} - (\alpha+\beta)\frac{T_0}{m_0}\sin\alpha_0 - \left(\frac{m}{m_0} - \frac{T}{T_0}\right)\frac{T_0}{m_0}\cos\alpha_0 \quad (9)$$

and

$$a_z = \left(\frac{m}{m_0} - \frac{L}{L_0}\right)\frac{L_0}{m_0} + \beta\frac{D_0}{m_0} - (\alpha+\beta)\frac{T_0}{m_0}\cos\alpha_0 + \left(\frac{m}{m_0} - \frac{T}{T_0}\right)\frac{T_0}{m_0}\sin\alpha_0 \quad (10)$$

and

$$\frac{d^2\theta}{dt^2} = \frac{M}{I} - \ddot{\theta}_0. \quad (11)$$

The analyst dealing with the dynamics of systems which inclued aerodynamic forces is usually faced with the conflict of deciding whether

to use the coordinates commonly employed by aerodynamicists or those conventionally used in mechanics. The root of this conflict rests in the fact that the variables most suitable for discussing the inertia forces are not especially convenient for the aerodynamic terms. However, the solution of the equations cannot depend on the choice of the coordinates and, therefore, it is simply a matter of preference which is to be employed. We shall use the nomenclature and variables common in mechanics and summarized in Table I.

TABLE I.

Coordinate Axes	Parallel to Coordinate Axes		About Coordinate Axes			
	Vel.	Accel.	Disp.	Vel.	Mom.	Inertia
x	u	a_x				
y			θ	q	M	B
z	w	a_z				

Accordingly, the perturbations in the accelerations are

$$\left.\begin{aligned} a_x &= \frac{du}{dt} + w_0\frac{d\theta}{dt} + w\frac{d\theta_0}{dt} \\ a_z &= \frac{dw}{dt} - u_0\frac{d\theta}{dt} - u\frac{d\theta_0}{dt} \\ a_\theta &= \frac{d^2\theta}{dt^2} \end{aligned}\right\}. \tag{12}$$

In addition, a change-over from the nomenclature used heretofore to that of Table I implies the definitions

$$\left.\begin{aligned} V_0 &\equiv u_0 \\ V &\equiv u \\ I &\equiv B \end{aligned}\right\} \tag{13}$$

and the relation

$$\bar{\alpha} = \frac{\bar{w}}{\bar{u}}. \tag{14}$$

When Eq. 14 is expanded in first order terms of small quantities, the perturbation in the angle of attack becomes

$$\alpha = \frac{w}{u_0} - \alpha_0\frac{u}{u_0} \tag{15}$$

and in view of the fact that $\theta = \alpha + \beta$, the angle of inclination of the flight path becomes

$$\beta = \frac{\alpha_0}{u_0} u - \frac{1}{u_0} w + \theta. \tag{16}$$

It will be observed that, now, the velocity components u and w, and the pitching displacement θ are the only dependent variables of this problem. When their time history and the initial flight conditions are prescribed, the entire trajectory and, therefore, all forces in the equations of motion are also defined in time; it follows that L, D, M, T and m are expressible in terms of u, w and θ. However, until the equations are solved, the time history of the dependent variables is not known, and use must be made of the fact that the mass, thrust, lift, drag and pitching moment depend also on the pitching velocity q, the air density ρ, the Mach number Ma and the elevator deflection δ. Among these, the air density and the Mach number take a special position. It will have been noticed that, in a formal sense, the solution to the differential equations requires initial values of u, w, θ and q only, but none on the position in space. However, the air density and Mach number do depend on the initial position because they are functions of the altitude. Therefore, the solution of our problem does require implicitly that the initial altitude be prescribed if, for instance, it is required to evaluate the trajectory perturbations which arise when the nominal trajectory was computed for a cold day and the effect of a change to a hot day is to be determined. The simplest way of dealing with this difficulty is to regard ρ and Ma as forcing functions and to class them with the elevator deflection perturbation δ.

Since every quantity without subscript in the variational equations represents a perturbation, it is seen that the equations permit now the treatment of small disturbances in all of them. However, the equations do not yet reflect the possibility that trajectory perturbations may arise from disturbances in initial conditions. In other words, the initial conditions on $\bar{u}$, $\bar{w}$ and $\bar{\theta}$ may not coincide precisely with the nominal initial conditions, and this disturbance alone will produce a neighboring trajectory. We provide for perturbations from this source by writing the perturbed initial values as

$$\left.\begin{aligned} \bar{u}(0) &= u_0(0) + u(0) \\ \bar{w}(0) &= w_0(0) + w(0) \\ \bar{\theta}(0) &= \theta_0(0) + \theta(0) \\ \bar{q}(0) &= q_0(0) + q(0) \\ \bar{\rho}(0) &= \rho_0(0) + \rho(0) \\ \bar{Ma}(0) &= Ma_0(0) + Ma(0) \end{aligned}\right\}. \tag{17}$$

Therefore, the mass and aerodynamic terms may now be written as

$$\left.\begin{aligned} L &= L(0)+L_u u+L_w w+L_\theta\theta+L_q q+L_\rho\rho+L_{Ma}Ma+L_\delta\delta \\ D &= D(0)+D_u u+D_w w+D_\theta\theta+D_q q+D_\rho\rho+D_{Ma}Ma+D_\delta\delta \\ M &= M(0)+M_u u+M_w w+M_\theta\theta+M_q q+M_\rho\rho+M_{Ma}Ma+M_\delta\delta \\ T &= T(0)+T_u u+T_w w+T_\theta\theta+T_q q+T_\rho\rho+T_{Ma}Ma+T_\delta\delta \\ m &= m(0)+m_u u+m_w w+m_\theta\theta+m_q q+m_\rho\rho+m_{Ma}Ma+m_\delta\delta \end{aligned}\right\} \quad (18)$$

where the subscripts denote partial differentiation and the general term

$$Q(0) = Q(u(0), w(0), \theta(0), q(0), \rho(0), Ma(0), \delta(0)). \quad (19)$$

In general, these latter quantities can be shown to have the property that $Q(0)=0$ when $u(0)=w(0)=\theta(0)=q(0)=\rho(0)=Ma(0)=\delta(0)=0$. An exception occurs in the case of the mass m. If time were always counted from the instant of flight initiation, rather than that of some (later) maneuver initiation, the quantity $m(0)$ would share with the others the property just demonstrated. This would be the case even though m, being a mass, is in quite a different class from the aerodynamic terms. However, it is often expedient to count time from the instant of maneuver initiation. For instance, if a maneuver, whose trajectory is the subject of interest, were preceded by a long cruising flight, one would hardly wish to consider the problem in the interval between flight and maneuver initiation. The basic reason why time may be counted from any instant on is that the aerodynamic terms depend only on the flight condition (altitude, velocity and direction) at the instant which is defined as initial, and not on the past flight. However, the mass—that is, the quantity of fuel remaining at maneuver initiation—does depend on the entire past flight history. But we may still admit the possibility of trajectory perturbations from uncertainties regarding the mass at the initial instant by supposing that $m(0)$ may or may not vanish when $u(0) = w(0) = \theta(0) = q(0) = \rho(0) = Ma(0) = \delta(0) = 0$. In every other respect, the mass acts just like the aerodynamic forces and moments because the rate of fuel consumption depends, in ramjet engines for instance, on altitude, velocity and angle of attack as do the aerodynamic terms. Thus, there are good reasons for grouping it with the aerodynamic forces.

If Eqs. 12 to 19 are substituted in the perturbation equations, the resulting equations may be compactly written as the matrix equation

$$[a_{ij}]\{\xi_i\} = [b_{ij}]\{\eta_j\} \quad (20)$$

where

$[\ ]$ denotes a square matrix,
$\{\ \}$ denotes a column matrix,
$i = 1, 2, 3, 4$ denotes a row,
$j = 1, 2, 3, 4$ denotes a column,

and the coefficients are listed in Appendix I.

The system of equations represented by Eq. 20 gives the perturbation equations in the most general form attempted in this paper. They do not, of course, possess complete generality. One obvious respect in which they depart from it is that they are restricted to flight in a vertical plane. Another lack of generality will be observed in the treatment of the pitching moment of inertia B. Fundamentally, there is no reason why this inertia should not be treated in the same manner as the mass, and treating it so would not introduce any profound difficulties, not already present in Eq. 20. The reason why the treatment was not extended to this point is that it is generally assumed (and we shall later substantiate this assumption) that the trajectory is quite insensitive to the magnitude of B. If we add the observation that B itself is in most aircraft quite insensitive to the rate of fuel consumption, there remains little reason to treat B as a variable which depends on perturbations in the dependent variables.

DISCUSSION OF THE EQUATIONS

The perturbation equations which we have derived are linear equations of motion of flight about a nominal trajectory. Therefore, they must contain as a special case the so-called equations of dynamic stability. In fact, these equations emerge when the nominal trajectory is taken to be one of straight, level uniform flight. One may, therefore, use the perturbation equations to investigate the stability of the nominal trajectory. If the flight is everywhere dynamically stable, but not critically or more than critically damped, the perturbed flight path will be a damped oscillation about the nominal trajectory. This result lends an appearance of unreality to the system of equations so far as the purpose in hand is concerned because it is our intention to investigate the effects of changes in the differential equations and initial conditions on the trajectory, not the stability of the nominal trajectory. As a result, we should expect a trajectory closely resembling the nominal one. For instance, if the nominal trajectory is not oscillatory, the neighboring one is not expected to be oscillatory either. Actually, this expected trajectory is also contained in the solutions of Eq. 20. In the presence of disturbances, that equation represents always a system of *inhomogeneous* linear equations and it is the particular solution to the inhomogeneous system which yields the trajectory for which we are searching. Thus, for our purpose, we shall ignore the general solution to the homogeneous system and consider the particular solution only.

Therefore, it must be shown that the system *is* always inhomogeneous if the nominal flight is not straight and level, and that the inhomogeneity does not depend on certain special perturbations. Let us write Eq. 20 as

$$\begin{aligned} a_{11}u + a_{12}w + a_{13}\theta + a_{14}q &= b_{11}\delta + b_{12}\rho + b_{13}Ma + b_{14} \\ a_{21}u + a_{22}w + a_{23}\theta + a_{24}q &= b_{21}\delta + b_{22}\rho + b_{23}Ma + b_{24} \\ a_{31}u + a_{32}w + a_{33}\theta + a_{34}q &= b_{31}\delta + b_{32}\rho + b_{33}Ma + b_{34} \\ a_{43}\theta + a_{44}q &= 0 \end{aligned} \tag{21}$$

where all symbols have the meaning introduced heretofore.

If the nominal trajectory is simply level uniform flight, $b_{i4}(t) \equiv 0$ for $i = 1, 2, 3$ if no perturbations of the initial values are contemplated. If none in δ, ρ and Ma are considered either, then $\delta = \rho = Ma \equiv 0$, and Eqs. 24 do become homogeneous. In fact, the well-known equations of the "stick-fixed" stability emerge. However, if the nominal trajectory is any kind of non-uniform flight, either accelerated straight, or curved in any manner whatever, the quantity $\theta_0 = \ddot{\alpha}_0 + \ddot{\beta}_0$ in b_{34} can never vanish identically, even if all initial perturbation values are zero, and if no perturbations on δ, ρ or Ma are admitted.

The fact that Eqs. 20 or 21 are linear is of little comfort as regards attempts at "closed form" or tabulated solutions. The difficulty lies in the fact that the a_{ij} and b_{ij} contain quantities with zero subscripts; these are functions of time which depend on the nominal trajectory and which are known in tabulated or graphical form, only. Therefore, solution of the perturbation equations may be expected to require the application of numerical methods.

SCHEME OF INTEGRATION

Basically, our method of dealing with the perturbation equations consists in considering the time dependent coefficients as constant during sufficiently short time intervals. Below we sketch out the validity of this procedure.

Consider, for instance, the first Eqs. 21. It may be written as

$$u = u(0) + \int_0^t (b_{11}\delta + b_{12}\rho + b_{13}Ma + b_{14} - a_{11}'u - a_{12}w - a_{13}\theta - a_{14}q)dt \tag{22}$$

where $a_{11}' = a_{11} - \dfrac{d}{dt}$ and all other symbols have been defined. The functions of time in the a_{ij} and b_{ij} are the quantities with zero subscripts obtained from the nominal trajectory. They (and their first derivatives if they occur) have at most isolated finite discontinuities; if it were otherwise, flight along the nominal trajectory would have

required infinite accelerations. Moreover, u, w, θ, q, ρ and Ma must be continuous everywhere if the trajectory is to have physical reality, and δ has at most isolated finite discontinuities (as in step function type of elevator control, for instance). Therefore, with Eq. 25 written as

$$u = u(0) + \sum_i \int_0^t f_i(t)x_i(t)dt \tag{23}$$

we are assured that the integrands are piecewise smooth and finite everywhere in the interval of integration. If the discontinuities in the interval $0 \leqslant t \leqslant t_s$ occur at the instants $t = t_k$, $k = 1, 2, \cdots, s-1$, we may evaluate Eq. 23 at $t = t_s$ as

$$u(t_s) = u(0) + \sum_i \left(\int_0^{t_1} f_i x_i dt + \sum_{r=1}^{s-1} \int_{t_r}^{t_{r+1}} f_i x_i dt \right) \tag{24}$$

and the integrands are now continuous and smooth everywhere in the intervals of integration. Therefore, any one of them may be written as

$$\int_{t_r}^{t_{r+1}} f_i x_i dt = \bar{f}_i \int_{t_r}^{t_{r+1}} x_i dt$$

where $\bar{f}_i$ is some average value of f_i at an instant τ in $t_r < \tau < t_{r+1}$.

It follows that in a sufficiently small time interval $\Delta t = t_{k+1} - t_k$ we may consider the general quantity with zero subscript $Q_0(t)$ to take on the value $Q_0(\tau) = Q_\tau$ where $\tau = \frac{1}{2}(t_{k+1} - t_k)$.

In the time interval $t_k \leqslant t \leqslant t_{k+1}$ the perturbation equations may now be considered as equations with constant coefficients. They are

$$\left.\begin{aligned} \left(\frac{d}{dt} + \alpha_{11}\right)u + \alpha_{12}w + \left(\alpha_{13}\frac{d}{dt} + \alpha_{14}\right)\theta &= \beta_1 \\ \alpha_{21}u + \left(\frac{d}{dt} + \alpha_{22}\right)w + \left(\alpha_{23}\frac{d}{dt} + \alpha_{24}\right)\theta &= \beta_2 \\ \alpha_{31}u + \alpha_{32}w + \left(\frac{d^2}{dt^2} + \alpha_{33}\frac{d}{dt} + \alpha_{34}\right)\theta &= \beta_3 \end{aligned}\right\} \tag{25}$$

where the coefficients are listed in Appendix II.

The quantities β_i have the character of forcing functions. In them, the elevator deflection δ, the air density ρ and the Mach number Ma are time-functions of perturbations magnitude; their coefficients and all other terms in the β_i are (considered) constant. The quantities δ, ρ and Ma are the differences in elevator deflection, air density or Mach number, respectively, between nominal and perturbed conditions. They are arbitrary in the sense that we are at liberty to specify what values they should take on, and it is they (besides initial value dis-

turbances or changes in parameters) which give rise to the perturbations. These variables are generally slowly varying functions of time. For instance, if one wishes to evaluate the effect of a hot day on a trajectory computed for a cold day, the quantities Ma and ρ are prescribed by the temperature differential and they vary slowly and monotonely with altitude. If effects on the trajectory due to uncertainties in the elevator program are to be computed, one might investigate such cases as $\delta = A\delta_0$ or $\delta = f(t)\delta_0$ where $f(t)$ is some slowly varying time function. Elevator programs of the sort $\delta = A \sin \omega t$ may be useful for studying the stability of the nominal trajectory; but they are excluded from our considerations because it is our purpose to compute trajectories which resemble the nominal ones in every respect, and oscillatory elevator motions will obviously not yield such flight paths. If the β_i are at most slowly varying time functions they may, during sufficiently short time intervals Δt, be considered to be constant at their mean values. Consequently, the particular solutions to Eqs. 25 are simply

$$u = \frac{\Delta_1}{\Delta}, \quad w = \frac{\Delta_2}{\Delta}, \quad \theta = \frac{\Delta_3}{\Delta} \tag{26}$$

where

$$\Delta_1 = \begin{vmatrix} \beta_1 & \alpha_{12} & \alpha_{14} \\ \beta_2 & \alpha_{22} & \alpha_{24} \\ \beta_3 & \alpha_{32} & \alpha_{34} \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} \alpha_{11} & \beta_1 & \alpha_{14} \\ \alpha_{21} & \beta_2 & \alpha_{24} \\ \alpha_{31} & \beta_3 & \alpha_{34} \end{vmatrix},$$

$$\Delta_3 = \begin{vmatrix} \alpha_{11} & \alpha_{12} & \beta_1 \\ \alpha_{21} & \alpha_{22} & \beta_2 \\ \alpha_{31} & \alpha_{32} & \beta_3 \end{vmatrix}, \quad \Delta = \begin{vmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{vmatrix}.$$

It is seen that the calculation of the perturbations proceeds by straightforeward algebraic means.

EXAMPLES

The solutions, given in Eqs. 26, possess considerable generality in that they yield the perturbed flight path under any disturbances which lie within the scope of the theory (so far as their identity and magnitude is concerned). In this theory, an attempt has been made to include perturbations in all parameters, functions, or initial conditions which enter the problem. The solutions may be used to evaluate perturbations in any one, or in any combination of some or all of them. It may be expected that the equations undergo significant simplifications when only a single source of flight perturbation is considered. We shall now discuss some examples.

One of the more interesting applications of the foregoing theory deals with considerations of the mass moment of inertia in pitch. Nominal trajectories are postulated to be those of a point mass missile;

in other words the pitching mass moment of inertia is taken to be zero. If the mass moment of inertia is restored from its "nominal" value $B_0 = 0$ to its physical value $\bar{B} = B_0 + B$ it is seen that the condition $|B| \ll |B_0|$ is violated. Thus, from a mathematical view point, there exists no good reason to expect that a first order theory in small quantities can be used to evaluate "perturbations" of this sort. From a physical point of view, one may well predict that the trajectory is not sensitive to the magnitude of the pitching moment of inertia. The reason for this is clear. The assumption that the moment of inertia is zero implies that the response of the aircraft to elevator control is instantaneous. If the actual response of the aircraft to such control is rapid, then the effect of the moment of inertia is indeed small. But aircraft, in general, are designed to respond rapidly to control surface motion, and if they did not, the design would be unsatisfactory. Therefore, one may well expect to find the predicted insensitivity of trajectories to changes in the magnitude of the moment of inertia from zero to the actual value.

For the particular example under consideration, the simplifying assumption was made that the moment-of-inertia-perturbation does not produce any changes in tangential velocity. This assumption is usually made when the dynamic stability of aircraft is investigated; it is probably in better accord with reality in ours, than in the stability theory, because our perturbed trajectory resembles the nominal one more closely than the oscillatory flight path of unstable (or stable) flight resembles the nominal trajectory, that is, straight level flight, from which it was computed. This simplification removes the first equation and the leading terms of the remaining two from Eqs. 25. It also implies that the sum of thrust and drag vanishes and, thus, all derivatives of T and D with respect to any of the dependent variables drop out of the α_{ij} and β_{ij}. The same is not true for the T and D-terms with τ-subscript because they are inherited from the nominal trajectory which may well be one of tangentially accelerated flight.

We may now pose the following problem: The nominal trajectory, computed for a point-mass missile, is such that the normal acceleration (or the angle of attack program) has been prescribed. What is the effect of non-zero moment of inertia on the trajectory?

Physically it is clear that there is no effect on the trajectory for, if the velocity is not altered (in accordance with our assumption) and either the normal acceleration or the angle of attack is prescribed at every instant, then the trajectory is prescribed. However, while there is no effect on the trajectory, the missile dynamics is affected.

From the moment equation of the nominal trajectory (see Eq. 3) it is known that, when $I_0 = B_0 = 0$,

$$M_0 = M_w w_0 + M_\vartheta \vartheta_0 + M_q q_0 + M_\delta \delta_0 = 0.$$

This equation is obtained with the aid of the third of Eqs. 21 with all zero-derivatives omitted. It follows that

$$\delta = \frac{1}{M_\delta}\,(M_w w + M_\theta \theta + M_q q)$$

and this is, in fact, the elevator perturbation required to achieve the prescribed program in the presence of non-vanishing moment of inertia. Substitutions of the last equation in the third of Eqs. 18 (with zero-derivatives omitted) shows that $M \equiv 0$. Substitution of this value in Eq. 8—that is, the "moment perturbation" equation—shows that the only solution which satisfies the initial conditions of the problem is $\theta = -\theta_0$. In words, the solution states that the missile must pitch through an angle such that the angle of attack is indeed the angle between body axis and flight direction. This was not achieved in the nominal trajectory because pitch was not one of the admitted degrees of freedom.

A problem more indicative of the effect of the moment of inertia on the trajectory is the following:

The nominal trajectory, computed for a point mass missile, is such that either the normal acceleration or the angle of attack program has been prescribed. This defines the nominal program. What is the effect of non-zero moment of inertia on the trajectory if the elevator program remains as computed for the point mass missile?

This problem has again been solved under the assumption that the tangential velocity is unaffected. Since the elevator program is the same as in the nominal trajectory, we have $\delta \equiv 0$, that is, no perturbations in δ are permitted. The solutions now take the form

$$w = \frac{\Delta_2}{\Delta}, \quad \theta = \frac{\Delta_3}{\Delta}$$

and the determinants are

$$\Delta_2 = \begin{vmatrix} \beta_2 & \alpha_{24} \\ \beta_3 & \alpha_{34} \end{vmatrix}, \quad \Delta_3 = \begin{vmatrix} \alpha_{22} & \beta_2 \\ \alpha_{32} & \beta_3 \end{vmatrix}, \quad \Delta = \begin{vmatrix} \alpha_{22} & \alpha_{23} \\ \alpha_{32} & \alpha_{33} \end{vmatrix}$$

where the determinant elements are

$$\alpha_{22} = \frac{L_w}{m_r} + \frac{D_r}{m_r u_r}, \quad \alpha_{32} = -\frac{M_w}{B}$$

$$\alpha_{24} = \frac{L_\theta}{m_r} - \frac{D_r}{m_r}, \quad \alpha_{34} = -\frac{M_\theta}{B}$$

$$\beta_1 = 0, \quad \beta_2 = -\frac{L(t_k)}{m_r}, \quad \beta_3 = \frac{M(t_k)}{B} - \vartheta_r.$$

As a final example, we demonstrate the application of Eqs. 26 to a temperature perturbation. The nominal trajectory is again that in which a prescribed angle of attack program is followed by a specified normal acceleration program, and it has been computed for a cold day. It is desired to compute the neighboring trajectory which results from the conditions on a hot day with the same flight program maintained and the moment of inertia effects ignored. Since the latter are excluded from consideration, the neighboring trajectory is also that of a point mass missile and the last of Eqs. 26 as well as the θ-terms in the first two vanish identically.

Since the angle of attack or the normal acceleration is prescribed at every instant of flight on both the nominal and perturbed trajectories, it is obvious that these trajectories must have different elevator programs because identical elevator programs under different flight conditions (such as different velocities) would result in different flight programs. Therefore, a perturbation in the elevator program will take place, and $\delta(t) \neq 0$. It follows that the determinant elements for the case under consideration are:

$$\alpha_{11} = \frac{D_w}{m_\tau} + \frac{L_\tau \alpha_\tau}{m_\tau u_\tau} - \frac{T_w}{m_\tau} \cos \alpha_\tau$$

$$\alpha_{12} = \dot{\theta}_\tau + \frac{D_w}{m_\tau} - \frac{L_\tau}{m_\tau u_\tau} - \frac{T_w}{m_\tau} \cos \alpha_\tau$$

$$\alpha_{21} = -\dot{\theta}_\tau + \frac{L_w}{m_\tau} - \frac{D_\tau \alpha_\tau}{m_\tau u_\tau} + \frac{T_w}{m_\tau} \sin \alpha_\tau$$

$$\alpha_{22} = \frac{L_w}{m_\tau} + \frac{D_\tau}{m_\tau u_\tau} + \frac{T_w}{m_\tau} \sin \alpha_\tau$$

$$\beta_1 = \left(-\frac{D_\delta}{m_\tau} + \frac{T_\delta}{m_\tau} \cos \alpha_\tau\right)\delta + \left(-\frac{D_\rho}{m_\tau} + \frac{T_\rho}{m_\tau} \cos \alpha_\tau\right)\rho + \left(-\frac{D_{Ma}}{m_\tau} + \frac{T_{Ma}}{m_\tau} \cos \alpha_\tau\right)Ma + \left(-\frac{D(t_k)}{m_\tau} + \frac{T(t_k)}{m_\tau} \cos \alpha_\tau\right)$$

$$\beta_2 = \left(-\frac{L_\delta}{m_\tau} - \frac{T_\delta}{m_\tau} \sin \alpha_\tau\right)\delta + \left(-\frac{L_\rho}{m_\tau} - \frac{T_\rho}{m_\tau} \sin \alpha_\tau\right)\rho + \left(-\frac{L_{Ma}}{m_\tau} - \frac{T_{Ma}}{m_\tau} \sin \alpha_\tau\right)Ma + \left(-\frac{L(t_k)}{m_\tau} - \frac{T(t_k)}{m_\tau} \sin \alpha_\tau\right).$$

However, although the elevator perturbation does not vanish identically, it is not arbitrary. In the first portion of the flight where the angle of attack is prescribed, $\delta(t)$ must be such that perturbations in the angle of attack vanish. The resulting equation of constraint on δ is obtained as follows: Since pitch is not one of the admitted degrees of freedom, the angle of attack becomes a function of the elevator

deflection or, Eq. 15 must be modified to read

$$\alpha = 0 = \frac{w}{u_0} - \alpha_0 \frac{u}{u_0} + \frac{\partial \alpha}{\partial \delta}\delta$$

where the zero on the left is a consequence of the fact that perturbations in α_0 are not admitted when the angle of attack is prescribed. The above equation is the desired equation of constraint.

$$\delta = \frac{1}{\partial a/\partial \delta}\left(\alpha_0 \frac{u}{u_0} - \frac{w}{u_0}\right).$$

When this value of δ is substituted in β_1 and β_2, terms in u and w arise which must be ordered with the other coefficients of u and w in the equations. If we let $\frac{\partial \alpha}{\partial \delta} = r$, the new determinant elements become

$$\bar{\alpha}_{11} = \alpha_{11} - \frac{\alpha_r}{ru_r}\left(-\frac{D_u}{m_r} + \frac{T_u}{m_r}\cos\alpha_r\right)$$

$$\bar{\alpha}_{12} = \alpha_{12} + \frac{1}{ru_r}\left(-\frac{D_\alpha}{m_r} + \frac{T_\alpha}{m_r}\cos\alpha_r\right)$$

$$\bar{\alpha}_{21} = \alpha_{21} - \frac{\alpha_r}{ru_r}\left(-\frac{L_u}{m_r} - \frac{T_u}{m_r}\sin\alpha_r\right)$$

$$\bar{\alpha}_{22} = \alpha_{22} - \frac{1}{ru_r}\left(-\frac{L_\alpha}{m_r} - \frac{T_\alpha}{m_r}\sin\alpha_r\right)$$

$$\bar{\beta}_1 = \beta_1(\delta = 0), \quad \bar{\beta}_2 = \beta_2(\delta = 0)$$

where the notation on the last two means that the $\bar{\beta}_{1,2}$ are the same as the $\beta_{1,2}$ respectively with δ set equal to zero. The perturbation solutions now become

$$u = \frac{\bar{\Delta}_1}{\bar{\Delta}}, \quad w = \frac{\bar{\Delta}_2}{\bar{\Delta}}$$

$$\bar{\Delta}_1 = \begin{vmatrix} \bar{\beta}_1 & \bar{\alpha}_{12} \\ \bar{\beta}_2 & \bar{\alpha}_{22} \end{vmatrix}, \quad \bar{\Delta}_2 = \begin{vmatrix} \bar{\alpha}_{11} & \bar{\beta}_1 \\ \bar{\alpha}_{21} & \bar{\beta}_2 \end{vmatrix}, \quad \bar{\Delta} = \begin{vmatrix} \bar{\alpha}_{11} & \bar{\alpha}_{12} \\ \bar{\alpha}_{21} & \bar{\alpha}_{22} \end{vmatrix}$$

applicable in that range of the trajectory where the angle of attack is prescribed.

When the normal acceleration is prescribed, rather than the angle of attack, a somewhat different equation of constraint on the elevator perturbation is obtained. If the normal acceleration is equal to n_0, the lift may be written as $L_0 = n_0 W_0$. With the inapplicable derivatives omitted, the first of Eqs. 21 yields

$$nW = 0 = L(t_k) + L_u u + L_w w + L_p p + L_{M_\alpha} M\dot{\alpha} + L_\delta \delta$$

where the zero appears because no perturbations in n are admitted. Solved for δ, this yields the equation of constraint

$$\delta = -\frac{1}{L_\delta}\left(L(t_k) + L_u u + L_w \bar{w} + L_\rho \rho + L_{Ma} Ma\right).$$

When this value of δ is substituted in β_1 and β_2, these values inserted in the equations

$$\alpha_{11} u + \alpha_{12} w = \beta_1$$
$$\alpha_{21} u + \alpha_{22} w = \beta_2$$

and like terms are collected, a new set of equations and determinant elements is obtained and new perturbation solutions result much as was done with the barred determinant elements.

It should be evident from these examples in what manner neighboring trajectories are computed when the perturbation arises from causes other than those considered in the above examples.

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APPENDIX I

The matrix elements of Eq. 20 are of central importance when the methods of this paper are used in actual computations. Therefore, they are listed below.

$$a_{11} = \frac{d}{dt} - \left(\frac{m_u}{m_0} - \frac{D_u}{D_0}\right)\frac{D_0}{m_0} + \frac{L_0 \alpha_0}{m_0 u_0} + \left(\frac{m_u}{m_0} - \frac{T_u}{T_0}\right)\frac{T_0}{m_0}\cos\alpha_0,$$

$$a_{12} = \dot{\theta}_0 - \left(\frac{m_w}{m_0} - \frac{D_w}{D_0}\right)\frac{D_0}{m_0} - \frac{L_0}{m_0 u_0} + \left(\frac{m_w}{m_0} - \frac{T_w}{T_0}\right)\frac{T_0}{m_0}\cos\alpha_0,$$

$$a_{13} = w_0 \frac{d}{dt} - \left(\frac{m_\theta}{m_0} - \frac{D_\theta}{D_0}\right)\frac{D_0}{m_0} + \frac{L_\theta}{m_0} + \left(\frac{m_\theta}{m_0} - \frac{T_\theta}{T_0}\right)\frac{T_0}{m_0}\cos\alpha_0 + \frac{T_0}{m_0}\sin\alpha_0,$$

$$a_{14} = -\left(\frac{m_q}{m_0} - \frac{D_q}{D_0}\right)\frac{D_0}{m_0} + \left(\frac{m_q}{m_0} - \frac{T_q}{T_0}\right)\frac{T_0}{m_0}\cos\alpha_0,$$

$$a_{21} = -\dot{\theta}_0 - \left(\frac{m_u}{m_0} - \frac{L_u}{L_0}\right)\frac{L_0}{m_0} - \frac{D_0 \alpha_0}{m_0 u_0} - \left(\frac{m_u}{m_0} - \frac{T_u}{T_0}\right)\frac{T_0}{m_0}\sin\alpha_0,$$

$$a_{22} = \frac{d}{dt} - \left(\frac{m_w}{m_0} - \frac{L_w}{L_0}\right)\frac{L_0}{m_0} + \frac{D_0}{m_0 u_0} - \left(\frac{m_w}{m_0} - \frac{T_w}{T_0}\right)\frac{T_0}{m_0}\sin\alpha_0,$$

$$a_{23} = -u_0 \frac{d}{dt} - \left(\frac{m_\theta}{m_0} - \frac{L_\theta}{L_0}\right)\frac{L_0}{m_0} - \frac{D_0}{m_0} - \left(\frac{m_\theta}{m_0} - \frac{T_\theta}{T_0}\right)\frac{T_0}{m_0}\sin\alpha_0 + \frac{T_0}{m_0}\cos\alpha_0,$$

$$a_{24} = -\left(\frac{m_q}{m_0} - \frac{L_q}{L_0}\right)\frac{L_0}{m_0} - \left(\frac{m_q}{m_0} - \frac{T_q}{T_0}\right)\frac{T_0}{m_0}\sin\alpha_0,$$

$$a_{31} = -\frac{M_u}{B}, \quad a_{32} = -\frac{M_w}{B}, \quad a_{33} = \frac{d^2}{dt^2} - \frac{M_\theta}{B}, \quad a_{34} = -\frac{M_q}{B},$$

$$a_{41} = a_{42} = 0, \quad a_{43} = \frac{d}{dt}, \quad a_{44} = -1,$$

$$\xi_1 = u, \quad \xi_2 = w, \quad \xi_3 = \theta, \quad \xi_4 = q,$$

$$b_{11} = \left(\frac{m_\delta}{m_0} - \frac{D_\delta}{D_0}\right)\frac{D_0}{m_0} - \left(\frac{m_\delta}{m_0} - \frac{T_\delta}{T_0}\right)\frac{T_0}{m_0}\cos\alpha_0,$$

$$b_{12} = \left(\frac{m_\rho}{m_0} - \frac{D_\rho}{D_0}\right)\frac{D_0}{m_0} - \left(\frac{m_\rho}{m_0} - \frac{T_\rho}{T_0}\right)\frac{T_0}{m_0}\cos\alpha_0,$$

$$b_{13} = \left(\frac{m_{Ma}}{m_0} - \frac{D_{Ma}}{D_0}\right)\frac{D_0}{m_0} - \left(\frac{m_{Ma}}{m_0} - \frac{T_{Ma}}{T_0}\right)\frac{T_0}{m_0}\cos\alpha_0,$$

$$b_{14} = \left(\frac{m(0)}{m_0} - \frac{D(0)}{D_0}\right)\frac{D_0}{m_0} - \left(\frac{m(0)}{m_0} - \frac{T(0)}{T_0}\right)\frac{T_0}{m_0}\cos\alpha_0,$$

$$b_{21} = \left(\frac{m_\delta}{m_0} - \frac{L_\delta}{L_0}\right)\frac{L_0}{m_0} + \left(\frac{m_\delta}{m_0} - \frac{T_\delta}{T_0}\right)\frac{T_0}{m_0}\sin\alpha_0,$$

$$b_{22} = \left(\frac{m_\rho}{m_0} - \frac{L_\rho}{L_0}\right)\frac{L_0}{m_0} + \left(\frac{m_\rho}{m_0} - \frac{T_\rho}{T_0}\right)\frac{T_0}{m_0}\sin\alpha_0,$$

$$b_{23} = \left(\frac{m_{Ma}}{m_0} - \frac{L_{Ma}}{L_0}\right)\frac{L_0}{m_0} + \left(\frac{m_{Ma}}{m_0} - \frac{T_{Ma}}{T_0}\right)\frac{T_0}{m_0}\sin\alpha_0,$$

$$b_{24} = \left(\frac{m(0)}{m_0} - \frac{L(0)}{L_0}\right)\frac{L_0}{m_0} + \left(\frac{m(0)}{m_0} - \frac{T(0)}{T_0}\right)\frac{T_0}{m_0}\sin\alpha_0,$$

$$b_{31} = \frac{M_\delta}{B}, \quad b_{32} = \frac{M_\rho}{B}, \quad b_{33} = \frac{M_{Ma}}{B}, \quad b_{34} = \frac{M(0)}{B} - \ddot{\theta}_0,$$

$$b_{41} = b_{42} = b_{43} = b_{44} = 0,$$

$$\eta_1 = \delta, \quad \eta_2 = \rho, \quad \eta_3 = Ma, \quad \eta_4 = 1.$$

APPENDIX II

When Eqs. 25 are put to actual use, it is essential to have expressions for the coefficients of these equations. The coefficients are listed below.

$$\alpha_{11} = -\left(\frac{m_u}{m_\tau} - \frac{D_u}{D_\tau}\right)\frac{D_\tau}{m_\tau} + \frac{L_\tau\alpha_\tau}{m_\tau u_\tau} + \left(\frac{m_u}{m_\tau} - \frac{T_u}{T_\tau}\right)\frac{T_\tau}{m_\tau}\cos\alpha_\tau,$$

$$\alpha_{12} = \dot{\theta}_\tau - \left(\frac{m_w}{m_\tau} - \frac{D_w}{D_\tau}\right)\frac{D_\tau}{m_\tau} - \frac{L_\tau}{m_\tau u_\tau} + \left(\frac{m_w}{m_\tau} - \frac{T_w}{T_\tau}\right)\frac{T_\tau}{m_\tau}\cos\alpha_\tau,$$

$$\alpha_{13} = -\left(\frac{m_q}{m_\tau} - \frac{D_q}{D_\tau}\right)\frac{D_\tau}{m_\tau} + w_\tau + \left(\frac{m_q}{m_\tau} - \frac{T_q}{T_\tau}\right)\frac{T_\tau}{m_\tau}\cos\alpha_\tau,$$

$$\alpha_{14} = -\left(\frac{m_\theta}{m_\tau} - \frac{D_\theta}{D_\tau}\right)\frac{D_\tau}{m_\tau} + \frac{L_\tau}{m_\tau} + \left(\frac{m_\theta}{m_\tau} - \frac{T_\theta}{T_\tau}\right)\frac{T_\tau}{m_\tau}\cos\alpha_\tau + \frac{T_\tau}{m_\tau}\sin\alpha_\tau,$$

$$\alpha_{21} = -\dot{\theta}_\tau - \left(\frac{m_u}{m_\tau} - \frac{L_u}{L_\tau}\right)\frac{L_\tau}{m_\tau} - \frac{D_\tau\alpha_\tau}{m_\tau u_\tau} - \left(\frac{m_u}{m_\tau} - \frac{T_u}{T_\tau}\right)\frac{T_\tau}{m_\tau}\sin\alpha_\tau,$$

$$\alpha_{22} = -\left(\frac{m_w}{m_\tau} - \frac{L_w}{L_\tau}\right)\frac{L_\tau}{m_\tau} + \frac{D_\tau}{m_\tau u_\tau} - \left(\frac{m_w}{m_\tau} - \frac{T_w}{T_\tau}\right)\frac{T_\tau}{m_\tau}\sin\alpha_\tau,$$

$$a_{33} = -\left(\frac{m_q}{m_\tau} - \frac{L_q}{L_\tau}\right)\frac{L_\tau}{m_\tau} - u_\tau - \left(\frac{m_q}{m_\tau} - \frac{T_q}{T_\tau}\right)\frac{T_\tau}{m_\tau}\sin\alpha_\tau,$$

$$a_{34} = -\left(\frac{m_\vartheta}{m_\tau} - \frac{L_\vartheta}{L_\tau}\right)\frac{L_\tau}{m_\tau} - \frac{D_\tau}{m_\tau} - \left(\frac{m_\vartheta}{m_\tau} - \frac{T_\vartheta}{T_\tau}\right)\frac{T_\tau}{m_\tau}\sin\alpha_\tau + \frac{T_\tau}{m_\tau}\cos\alpha_\tau,$$

$$a_{31} = -\frac{M_w}{B_\tau}, \quad a_{32} = -\frac{M_w}{B_\tau}, \quad a_{33} = -\frac{M_q}{B_\tau}, \quad a_{34} = -\frac{M_\vartheta}{B_\tau},$$

$$\beta_1 = \left[\left(\frac{m_\delta}{m_\tau} - \frac{D_\delta}{D_\tau}\right)\frac{D_\tau}{m_\tau} - \left(\frac{m_\delta}{m_\tau} - \frac{T_\delta}{T_\tau}\right)\frac{T_\tau}{m_\tau}\cos\alpha_\tau\right]\delta$$
$$+\left[\left(\frac{m_\rho}{m_\tau} - \frac{D_\rho}{D_\tau}\right)\frac{D_\tau}{m_\tau} - \left(\frac{m_\rho}{m_\tau} - \frac{T_\rho}{T_\tau}\right)\frac{T_\tau}{m_\tau}\cos\alpha_\tau\right]\rho$$
$$+\left[\left(\frac{m_{Ma}}{m_\tau} - \frac{D_{Ma}}{D_\tau}\right)\frac{D_\tau}{m_\tau} - \left(\frac{m_{Ma}}{m_\tau} - \frac{T_{Ma}}{T_\tau}\right)\frac{T_\tau}{m_\tau}\cos\alpha_\tau\right]Ma$$
$$+\left[\frac{m(t_k)}{m_\tau} - \frac{D(t_k)}{D_\tau}\right]\frac{D_\tau}{m_\tau} - \left[\frac{m(t_k)}{m_\tau} - \frac{T(t_k)}{T_\tau}\right]\frac{T_\tau}{m_\tau}\cos\alpha_\tau,$$

$$\beta_2 = \left[\left(\frac{m_\delta}{m_\tau} - \frac{L_\delta}{L_\tau}\right)\frac{L_\tau}{m_\tau} + \left(\frac{m_\delta}{m_\tau} - \frac{T_\delta}{T_\tau}\right)\frac{T_\tau}{m_\tau}\sin\alpha_\tau\right]\delta$$
$$+\left[\left(\frac{m_\rho}{m_\tau} - \frac{L_\rho}{L_\tau}\right)\frac{L_\tau}{m_\tau} + \left(\frac{m_\rho}{m_\tau} - \frac{T_\rho}{T_\tau}\right)\frac{T_\tau}{m_\tau}\sin\alpha_\tau\right]\rho$$
$$+\left[\left(\frac{m_{Ma}}{m_\tau} - \frac{L_{Ma}}{L_\tau}\right)\frac{L_\tau}{m_\tau} + \left(\frac{m_{Ma}}{m_\tau} - \frac{T_{Ma}}{T_\tau}\right)\frac{T_\tau}{m_\tau}\sin\alpha_\tau\right]Ma$$
$$+\left[\frac{m(t_k)}{m_\tau} - \frac{L(t_k)}{L_\tau}\right]\frac{L_\tau}{m_\tau} + \left[\frac{m(t_k)}{m_\tau} - \frac{T(t_k)}{T_\tau}\right]\frac{T_\tau}{m_\tau}\sin\alpha_\tau,$$

$$\beta_3 = \frac{M_\delta}{B_\tau}\delta + \frac{M_\rho}{B_\tau}\rho + \frac{M_{Ma}}{B_\tau}Ma + \frac{M(t_k)}{B_\tau} - \ddot{\vartheta}_\tau.$$

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THE FRANKLIN INSTITUTE

PRESENTATION OF THE VERMILYE MEDAL

Wednesday, April 2, 1958

The Vermilye Medal was presented to Frederick C. Crawford, Chairman of the Board, Thompson Products, Inc., Cleveland, Ohio, at a Special Meeting of The Franklin Institute held April 2, 1958.

The proceedings of that meeting follow.

* * *

MR. WYNN LAURENCE LEPAGE (*President, The Franklin Institute*): "Ladies and Gentlemen, this is a Special Meeting of The Franklin Institute, special because it is not one of the regular stated meetings of the Institute and special because we have come here this evening for a very special purpose.

"The holding of this meeting was formally approved by the Board of Managers at its last meeting, acting under Article XI, Section 3 of the By-Laws, for the specific purpose of presenting the Vermilye Medal Award for 1957.

"The Award was authorized by the Board of Managers of The Franklin Institute in a meeting in November, 1957 and, the recipient being an extremely busy and active man, it has taken us this time to get our net around him.

"The Franklin Institute, as you know, holds regular meetings at which we bring to you, Members of the Institute and friends, the great scientists and engineers of the age who present to us their accomplishments, lectures on their works and their achievements. One of these meetings, the regular Stated Meeting in October each year, is devoted to the presentation of the great Franklin Institute medals. These cherished awards are given to outstanding scientists and engineers throughout the world for achievement in the mechanical and scientific arts. This day in October is known as Medal Day.

"This evening we have another medal of The Franklin Institute to award, a medal which in the accepted dictionary definition doesn't come under quite the term 'science,' though we are convinced that the practice of the arts in which this evening's medalist is skilled is indeed

a great science; and I feel sure that when we have heard him we will come to this realization.

"To tell you about the Vermilye Medal and to introduce to you the one to whom we would present it, I call upon Mr. Francis J. Chesterman, member of the Board of Managers of The Franklin Institute, and Chairman of its Special Committee on the Vermilye Award.

"Mr. Chesterman."

MR. FRANCIS J. CHESTERMAN: "Mr. President, Ladies and Gentlemen: It is my privilege as Chairman of the Vermilye Medal Committee to tell you about the Medal and to present to you the recipient of this award.

"The founder of this medal, William H. Vermilye, was a great friend of The Franklin Institute, and because of this fact and because he wished to recognize outstanding contributions in the field of industrial management, he made this medal possible.

"This, as your President has said, is the only medal which The Franklin Institute awards which is not for scientific invention or the development of processes and theories.

"Mr. Vermilye believed that the functions of management are so fundamental to the development of this country's industry that he wished to honor those who were specially qualified as industrial managers. Mr. Vermilye, of Dutch descent, was a true American and staunch in his beliefs in the things which we in this country hold dear.

"He graduated from the Massachusetts Institute of Technology in June of 1901, but as I did not enter that institution until September of that year, I was not privileged to know him in those years. However, I did know and appreciate him in later years. Upon graduation, Mr. Vermilye went into the textile business, which at that time was being very rapidly expanded and developed, and being one of the pioneers, gave it most successful leadership. In fact, he developed in my judgment a balanced idea as between worker and management on a scale which had never been attempted heretofore.

"Toward the end of his career he became a Vice President of the National City Bank of New York, but never lost interest in industrial management problems. Moreover, he continued always to give his affection to The Franklin Institute.

"The Vermilye Medal Committee for 1957 was composed of Edward G. Budd, Jr., President of The Budd Company; James Creese, President of the Drexel Institute of Technology; Charles Penrose, Senior Vice President for North America for The Newcomen Society. Unfortunately, none of those gentlemen could be with us tonight. And I am the Chairman.

"I am sure that you will be interested to know to whom the Medal has been awarded in previous years. The first Medal was awarded to Mr. Louis H. Brown in 1939, at that time President of the Johns-

Manville Corporation. In 1941 the Medal was awarded to Mr. William S. Knudson, amongst other things, Head of the Office of Production Management in the critical period of World War II. In 1943 the Medal was presented to Mr. Walter S. Gifford, then President of the American Telephone and Telegraph Company, under whom it had been my pleasure to have served and who later became United States Ambassador to Great Britain.

"The next one to receive the Medal, in 1946, was Mr. Martin W. Clement, former President of the Pennsylvania Railroad and a most active man in Philadelphia. I am delighted that Mr. and Mrs. Clement are with us this evening.

"The next Medal was presented to Mr. J. Howard Pew, in 1950, Chairman of the Sun Oil Company, who had been such a factor in the growth and development of the oil industry of our country and who is vitally interested in many philanthropies. I am delighted also that Mr. and Mrs. Pew are with us this evening.

"The last Vermilye Medal was awarded in 1953 to Mr. Benjamin F. Fairless, then Chairman of the Board of United States Steel Corporation, who was active in the development of that industry in our northeast section and, of course, through that development has brought tremendous growth to this Port of Philadelphia.

"And now it is my privilege to present the 1957 Medalist to you. The award is made, as your President said, in 1958 because of the inability of our Medalist to be present at a prior time.

"Born a Yankee in Watertown, Massachusetts, he graduated from Harvard and upon graduation became a millwright helper in 1916 with the Steel Products Corporation, which later became Thompson Products, Inc.

"Typical of the man whom we honor is the fact that from this beginning he became a Director of the company in 1926, President in 1933, and Chairman of the Board since 1953. There I think we see the reason for his success. He early learned to know people, what their reactions were, how to work with them, and how to have them work for him.

"His knowledge and capacity were such that he was a management member of the Management-Labor Policy Committee of the Manpower Commission during World War II. He was on the President's War Advisory Council for Businessmen, and has served in many other capacities along similar lines.

"As a resident of Cleveland, he has filled varied and important positions both in civic and philanthropic lines. He has been awarded many medals and has been honored by several learned institutions, but I think he will be most remembered for his humanities and for his understanding of human nature and, as such, has become an industrial leader whom we are glad to honor.

"I will now read the citation for the award:"

> When an industrial leader, within whatever field, achieves personal and corporate success through exercise of vision, foresight, courage, resourcefulness, character and, above all, ceaseless and tireless hard work, then to that man is due high recognition. To such a man is America beholden in the restless progress of our nation's industrial advance.

"Moreover, to such a man living in Cleveland, Ohio, is the present Vermilye Award given. His has been vouchsafed opportunity through his own leadership and contagious inspiration to achieve for Thompson Products the measure of widely diversified contributions to American industry both in wartime and in peace.

"His is a life work which all of us properly may acclaim and may recognize.

"Mr. President, I present Mr. Crawford for this Award."

MR. LEPAGE: "Mr. Crawford, by virtue of the power vested in me as President of The Franklin Institute, it is my great privilege, sir, to present you with the Vermilye Medal Award for 1957 and the certificate which accompanies it, and my very warmest congratulations.

"Ladies and Gentlemen, Mr. Crawford."

MR. FREDERICK C. CRAWFORD: "Mr. President, Mr. Chesterman, Ladies and Gentlemen: Of course, I am very proud and very honored to receive this distinguished Medal and I accept it humbly. I know very many industrialists who deserve it more than I. I like to think, perhaps it is given rather for industrial achievement by all industry rather than to an individual, and on that basis I will accept it on behalf of many industrialists who have contributed to this great industry that we are so proud of."

* * *

Mr. Crawford then spoke on "An Experiment in Human Relations."

In his inspiring address, Mr. Crawford discussed a 35-year experiment in human relations at Thompson Products, wherein management and labor openly discuss their problems at mass meetings. He explained that this experience showed that "when men and management stand together in agreement, you don't need labor laws, lawyers or unions."

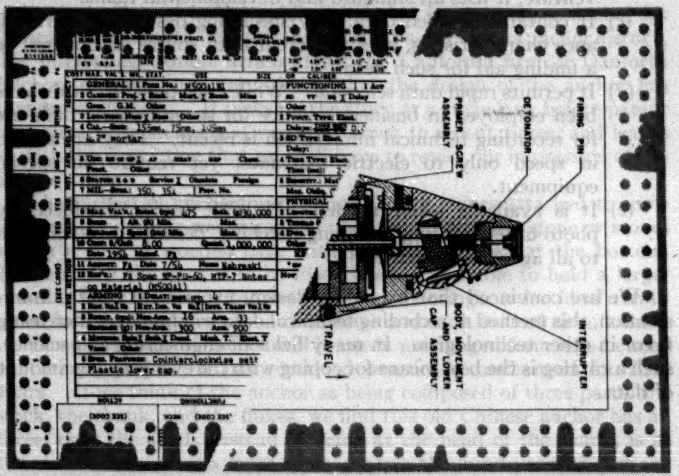
THE FRANKLIN INSTITUTE LABORATORIES
NICOL H. SMITH, DIRECTOR

TECHNICAL DATA ON BORDER-PUNCHED CARDS*

BY
GUNTHER COHN[1]

A versatile card catalog has been devised to serve as a readily accessible store of technical data. With over 100 bits of information per card, it is much more useful than a mere reference file.

Hand-sorted cards were chosen because they are easy to use and permit cross-reference in many ways. Such cards are border-punched to allow selection of key data by needle sorting. The file consists of 5″ by 8″ cards, with one card for each discrete item.



The Card contains Technical Data on the Front and a Sketch on the Back.

Convenient features were achieved by use of the various tricks of the trade on a card of new design. For example, it was apparent that space would not permit direct sorting (one item per hole) of all data. Hence, capacity was enlarged by use of more elaborate sorting methods (shallow, deep, and intermediate notches) for data that are sorted only rarely.

* From paper 58-SA-27 presented at the Semi-Annual meeting of The American Society of Mechanical Engineers, Detroit, Michigan, June 1958. This work was performed at The Franklin Institute under contract with Picatinny Arsenal.

[1] Senior Research Engineer.

To prevent an operator with a thin memory from erring, the meaning of notches is printed on the borders and a Guide with full instructions for use and explanations of all entries accompanies the file.

Benefits of such a file to designer and searcher are obvious: they will have at their fingertips a full record of previous ideas. Furthermore, supervisor and analyst can determine custom, know-how, and trend.

Our example of such a file is the Catalog of Ordnance fuzes and their explosive components. Each card contains technical data on the front and a sketch on the back. The Catalog is unique in several ways:

(*a*) It is one of the first technical compilations. Specifically, it consists of technical and military fuze data.

(*b*) It has complete coverage. A joint Army, Navy, and Air Force venture, it lists all standard and developmental items.

(*c*) It contains detailed data. By specifying design and performance characteristics, it is an actual data source as opposed to a finding aid for such data.

(*d*) It permits rapid data selection. While edge punched cards have been employed in business practice for several years, their use for recording technical information is recent. They are second in speed only to electric machines, yet require no special equipment.

(*e*) It is available where needed. Multiple copies, produced by photo-offset printing and gang notching, permitted distribution to all agencies concerned.

We are convinced that by aiding design, experiment, and administration, this method of recording technical data will find ever increasing favor in other technologies. In many fields of engineering and science, such a catalog is the best means for coping with the ever growing amount of data.

ANCHORS

BY
THOMAS COULSON [1]

With the co-operation of the U. S. Patent Office our Museum has been able to display two more interesting exhibits demonstrating how industries have been built up behind the protection afforded by the patent law. One of the exhibits relates to agricultural instruments and the other to anchors, showing that the patent law covers a wide field of industries.

At first glance there would not seem to be much likelihood of patents in the design of anchors. The shape of the common or kedge anchor is so firmly implanted in the public mind that it serves as a symbol for the sea and seafarers. By presenting a bird's-eye view of anchor history, this interesting exhibit arranged by the California firm of Danforth Anchors demonstrates very clearly that the design of the anchor has changed materially. It picks up the story at a stage long before patent laws were in existence and carries it down to recent times, and leaves us with the conviction that we shall have to revise our impression of the anchor's shape.

The earliest of anchors were simply heavy stones attached to some form of rope. This was improved upon by putting the stone or stones within a sort of wooden cage with projecting crooks at the bottom. This enabled the anchor to be made heavier and able to hold a larger vessel. They were not too inefficient as moorings under ideal conditions, but they dragged on hard bottoms and sank in mud or soft sand.

The oldest type of anchor still in use in unchanged form is to be seen on Chinese junks. It has remained practically unchanged for 2000 years. If we think of the anchor as being composed of three parts, the stock, the shank, and the flukes, we find this old Chinese anchor has all three, only the stock, instead of being at the head of the shank, is at the rear. Now, after a lapse of nearly two thousand years that is the position of the stock in the most recent anchor design in the United States.

The familiar anchor with two hooked arms and opposing stock appears on Greek coins issued twenty-seven hundred years ago. This anchor came into use in the Nile Delta where heavy stones were buried in the deep mud and could not be retrieved. An all-iron anchor would have suffered the same fate, so that they were sheathed in wood to prevent them from settling too deeply into the mud. The Romans were using an anchor similar to that of our familiar design in the first century.

[1] Director of Museum Research.

This had a wooden shank, two wooden arms iron tipped, and a lead stock. All-metal iron anchors were not unknown to the Roman mariners, and they followed the same design.

The common anchor with flukes and a wooden stock was introduced in the fifteenth century and remained standard equipment in the American Navy until 1860. In 1821 a patent was issued on the stockless anchor which is still used on some of the larger vessels. Its great advantage is the convenience of stowage in the hawse pipe.

A booklet distributed at the exhibit gives some interesting information on anchoring, which shows there is more to it than the choice of the anchor itself. Wind, current drag, and wave action have to be calculated before the load can be determined; the nature of the bottom where the anchor is to grip has to be considered; the nature of the cable has to be decided. All these and other features have to receive the attention of the careful amateur; the professional need not be reminded of them. The detail and variety of these factors will astonish the landlubber who would probably be reasonably satisfied by heaving overboard a heavy weight attached to a substantial rope.

The most significant feature about the modern anchor is the reduction in weight achieved without sacrificing any holding power. One of these modern scientifically designed anchors weighing only eight pounds has the same holding power as a common or kedge anchor of sixty pounds. The advantage in stowing away the anchor on deck is obvious.

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor. 472 items have been added the past month.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A.M. until 5 P.M. on Mondays, Tuesdays, Thursdays, and Fridays; 2 P.M. until 10 P.M. on Wednesdays; and from 9 A.M. until Noon on Saturdays.

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BOOK REVIEWS

LOGICAL DESIGN OF DIGITAL COMPUTERS, by Montgomery Phister, Jr. 408 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1958. Price, $10.00.

The province of activity for the logical designer of a digital computer lies between the system analysis and the circuit design. There are certain tools and techniques available to the logical designer in performing his duties and this book is devoted to their discussion. Provided these ingredients are mixed with a certain amount of creative genius, according to the author, the contents of the book are sufficient for one to perform the logical design of a digital computer.

The book is well written in text-book fashion and cannot be read lightly—it must be studied. This is not a book for someone totally unfamiliar with computers who wishes to be introduced to the field of machine computation. Rather, it is suited for those intimately engaged in the development end of computers. It is also recommended to computer personnel who use computers as a finished tool, such as programmers or numerical analysts; not because it will advance them in their own spheres of activity—it probably will not—but because it will give them an insight into an area too often taken for granted. Indeed, the computer user may well be so amazed at what he reads of logical design that he will gain an entirely new respect for the machine which has ceased to hold his fascination.

Of particular interest is the manner in which Mr. Phister has presented and developed the Boolean algebra. Every theorem, or its dual, is proved formally and then demonstrated by use of diode circuits and Venn or Veitch diagrams. Consequently, the theorems are readily appreciated and understood. Several different methods of simplifying Boolean functions are presented, including Quine's method, the Harvard method and Veitch diagram methods.

One chapter of the book is presented on the development of memory element input equations. The techniques are general for synchronous circuits but are of no help in the analysis of asynchronous circuits which operate independently of clock pulses. In fact the techniques described throughout the book are not applicable to the Princeton type machines, whose memory and decision circuits are asynchronous.

Another chapter is devoted to the derivation of application equations in which the Huffman-Moore digital computer model is used. The remainder of the book is concerned with computer memories, input-output equipment, the arithmetic unit and the control unit. Finally, two computers of a relatively simple nature are designed in full.

At the end of each chapter a variety of problems are provided for the reader. Unfortunately, however, the answers to these problems are not included. The extensive bibliographies at the end of each chapter indicate that Mr. Phister has drawn his material from various and recent sources, from men well qualified in the field, and he has successfully woven the many ideas and techniques of logical design into a worthy contribution.

LAURENCE C. MCGINN
The Franklin Institute Laboratories

BEHIND THE SPUTNIKS, by F. J. Krieger. 380 pages, diagrams, 6 × 9 in. Washington, Public Affairs Press, 1958. Price, $6.00.

In 1956 and 1957 this reviewer received Parts I and II of a series entitled "A Casebook on Soviet Astronautics." These contained papers and articles which indicated the scope of Soviet activity in the space travel field. Upon mature reflection, one cannot help but be amazed at our complacency in light of what the Russians were telling us. Yet with all this, our satellite efforts were retarded until we have reached the point where it may take years before we catch up with the Russians in the propulsion field.

We Americans who pride ourselves on our ability, our achievements and our aspirations, will do well to read this revealing work. In

Behind the Sputniks we find Soviet science laying the foundations for investigations on satellites, atomic airplanes, space stations, intercontinental ballistic missiles, biological aspects of space travel, travel to the planets, satellite telemetry and position finding and communications.

There is a special section on "Experimental Verification of the General Theory of Relativity and Artificial Earth Satellites," written by V. L. Ginzburg, which is especially revealing in that for the first time this reviewer has seen a treatment of another consequence of the General Theory. This involves the additional displacement of the perigee of a satellite or perihelion of a planet caused by the rotation of the central body. He shows that the advance in perigee of a satellite can amount to 50 seconds of arc a century—approximately the same as the effect of the General Theory of Relativity for Mercury!

Dr. Krieger has divided the book into seven sections. The section headings are indicative of the breadth of the Soviet approach to space travel. They are: Space Flight Comes of Age, Problems of Astronautics, Biological Factors, Lunar and Cosmic Projects, Rocket and Missile Developments, Satellite Plans, and The Sputnik.

The last section consists of the two reports from TASS and PRAVDA after the launching of the first Sputnik. Several appendices are included which yield significant information concerning purely Russian affairs. A complete bibliography is also included.

In the presentation of this work, Dr. Krieger and the Public Affairs Press have performed a notable service to this country. Now the layman can get a picture of pre-Sputnik plans of the Soviets to better understand the position of this country in the race for outer space.

I. M. LEVITT
The Fels Planetarium of The Franklin Institute

AIRCRAFT HYDRAULICS, edited by H. G. Conway. Volume I: Hydraulic Systems, 146 pages, diagrams, 6 × 9¼ in.; Volume II: Component Design, 198 pages, diagrams, 6 × 9¼ in. New York, The Macmillan Co., 1958. Price: Vol. I, $7.00; Vol. II, $9.00.

The first book of a series designed to describe the state of the art of aircraft design, *Aircraft Hydraulics*, Volume I, is the best advance advertising for the series. It is fitting that the first volume of such an undertaking be devoted to the systems which put the operating musculature in modern aircraft.

The presentation in Volume I is logical, starting with descriptions of various fluids used in hydraulic systems and progressing through hydraulic theory, piping, simple hydraulic circuits, aircraft installations, and system testing. The approach is general in the chapters covering hydraulic circuits, leaving the detailed description of the components for Volume II. Even so, Volume I is capable of giving the fledgling engineer an operating knowledge of the types of systems in use and acquainting him with the terminology of the field.

In Volume II the hydraulic system is considered on the basis of individual components. Chapters deal with seals, pumps and motors, selectors, valves and servos. A chapter dealing with miscellaneous components gives attention to such parts as accumulators, gauges, filters, and reservoirs or sumps. Each chapter describes the function of the hardware with which it is concerned in a clear, concise fashion, gives the basic design requirements for the component, and then describes the various component designs which have been used or are currently in use in the field.

Since the series of texts is sponsored by the Royal Aeronautical Society, it might be expected that most of the designs discussed would be of British origin. This is manifestly not the case. The authors of the individual chapters are expert in the field which they cover and are acquainted with American designs as well. It is interesting to note that in these two volumes an undercurrent of design standardization between British and U. S. designs is evident. In view of the international nature of modern transport and military aircraft such standardization becomes an important logistics point and certainly should not be overlooked in systems design.

The two texts are highly recommended for the aircraft engineer or for the student. The volumes are typical of British thoroughness and readability and form a basic compilation of twenty-odd years of experience in aircraft hydraulic systems design, maintenance, and operation.

RICHARD HOLLINGER
The Franklin Institute Laboratories

NUMERICAL CONTROL SYSTEMS FOR MACHINE TOOLS. 106 pages, illustrations, 8½ × 11 in. New York, Engineering Publishers, Inc., 1957. Price, $5.00 (paper).

Last September the Engineering Department of the Electronic Industries Association sponsored a symposium in Los Angeles devoted to reviewing the status of numerical control. Participants who presented papers at the symposium were largely staff members of electronics firms and machine tool manufacturers.

Those who are familiar with this field will recognize the fluid condition of control engineering and that a degree of standardization has not yet been achieved. Consequently many different schemes of control were discussed at the symposium. These ranged in complexity from simple numerical positioning to more elaborate systems for contour milling. Various phases of the work such as tape preparation and the terminology associated with this data medium were presented by proponents of the different systems.

These papers should certainly be skimmed by those who wish to be informed about the latest thinking in the numerical control field. The data on specific control systems are however summarized in a more complete and systematic way in the *Control Engineering* magazine series of January, February, March 1958, which should be consulted by those readers who are interested in specifications.

C. W. HARGENS
The Franklin Institute Laboratories

COMBINATION OF OBSERVATIONS, by W. M. Smart. 253 pages, diagrams, 5½ × 8½ in. New York, Cambridge University Press, 1958. Price, $6.50.

This text is mainly devoted to a discussion of how to treat fallible data in the physical sciences. To the extent that the rules of statistical inference and the rules for manipulating results are set forth, this book adds very little to an already crowded field. A chapter is devoted to the various techniques for representing statistical distributions by mathematical functions, and although the material is not new, the presentation is clear and convenient.

Chapter 6, which is devoted to equations of condition in several unknowns, provides information not generally found in statistics texts. The general problem is: given m linear equations, a measured quantity of known precision, and n independent variables in each equation where $n < m$, what is the most probable value of each of the n variables and how precise is this estimate?

In summary, this text places more emphasis on the fitting of models to data than the usual statistics book, and as such is more pertinent for students of the natural sciences. On the debit side, the wealth of statistical techniques for assessing data and drawing conclusions from data are scarcely mentioned.

E. S. KRENDEL
The Franklin Institute Laboratories

THE PLANET JUPITER, by B. M. Peek. 283 pages, plates, 5½ × 8½ in. New York, The Macmillan Co., 1958. Price, $8.50.

When this reviewer started his serious work in astronomy all the planets combined rated but a single chapter in an astronomical text book and of this chapter Mars took up about half. Today, definitive books have been written about Venus, Mars and now Jupiter.

In this new book written by Bertrand M. Peek, a fellow of the Royal Astronomical Society and for 15 years the Director of the Jupiter Section of the British Astronomical Society, for the first time a comprehensive survey of the planet is available. Among the various planet sections of the British Astronomical Society are found skilled, assiduous, dedicated observers of their specialties to the almost complete exclusion of other bodies. Thus these men have acquired a first hand detailed knowledge of their planets and they write with unquestioned authority. Such a book is "The Planet Jupiter."

The author has outlined this book masterfully.

In Part I is found the background for intelligent understanding of the planet. Its vital statistics are revealed and the nomenclature used in the book is defined.

Part II concerns observations of the planet's surface. Included is a brief resume of optical systems and observations with these. The author includes a discussion of color with the suggestions that true colors can only be obtained photographically with the use of filters. The remainder of this section is devoted to a

description of the entire surface of the planet with an extended section being devoted to the Great Red Spot. The inclusion of radio signals from Jupiter are treated with some background on the historical discovery.

Theoretical considerations constitute Part III. These deal with the physical and chemical composition of the interior. Models of the planet are given and these are based on some fundamental work done by W. H. Ramsey between 1948 and 1952. The author discusses the atmosphere and presents the work of the current authorities in the field.

The last section concerns the satellites of Jupiter. Some historical background is given. Included is a table with the statistics of the satellites. The author covers the occultation and eclipse phenomena involving the satellites.

"The Planet Jupiter" is a well written, excellently organized book which fills a pressing gap in the astronomer's library and is destined to be the reference book on the planet.

I. M. LEVITT
The Fels Planetarium of
The Franklin Institute

BOOK NOTES

RECENT ADVANCES IN THE ENGINEERING SCIENCES. 257 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $4.75.

This is the proceedings of a symposium held at Purdue University in September 1957 for deans of engineering. The conference dealt with the technical aspects and educational implications of such areas as automation, operations research, heat transfer, nuclear engineering, solid state physics and computers. Seven technical subjects were covered, each by two speakers—one giving a survey of the field and the other discussing its impact on engineering education. One paper was presented dealing with the high costs of engineering education. The speakers were all recognized authorities and their published remarks should be of real interest to all concerned with any phase of engineering education.

FINITE QUEUING TABLES, by L. G. Peck and R. N. Hazelwood. 210 pages, 8½ × 11 in. New York, John Wiley & Sons, Inc., 1958. Price, $8.50.

These extensive tables, computed on Univac, are published in the hope that they will prove helpful in the application of queuing theory to operating problems. The tables are for finite populations and offer great accuracy in dealing with such problems as communication channel requirements, machine loading determinations, and estimating requirements for both equipment and manpower. A ten-page preface gives examples of the types of problems for which the tables are applicable, and includes the equations used in their solution. The tables were offset directly from the Univac machine printing and so contain no human errors.

AIR CONDITIONING AND REFRIGERATION, by William H. Severns and Julian R. Fellows. 563 pages, illustrations, 6 × 9 in. New York, John Wiley & Sons, Inc., 1958. Price, $10.25.

Written principally as a textbook for undergraduate courses in mechanical engineering and architecture, this work will also be of considerable value to engineers in this field. Examples of typical calculations are given throughout, along with pictures of commercial equipment. One notable feature of this volume is the discussion of the complete procedure for the design of an air washer. Many comprehensive examples and problems are included on the design of hot water heating systems, fan-duct systems, and all-year air conditioning systems.

ENGINEERING ELECTROMAGNETICS, by William H. Hayt, Jr. 328 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., 1958. Price, $8.75.

In this clearly written introductory volume on the electric and magnetic fields, the primary objective is the development and understanding of Maxwell's equations. The author combines such techniques as vector analysis,

relativity and solid-state physics into a logically developed, unadorned presentation of electromagnetics at the junior level. The sections on relaxation and iteration methods of experimental mapping are a special feature of this volume. An appendix on dimensions and dimensional analysis is also included.

INTRODUCTION TO ELECTROMAGNETIC FIELDS, by Samuel Seely. 308 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $8.50.

The material in this introductory text is presented in order of increasing complexity of field concepts, beginning with the electric field in multidimensional conductors and ending with Maxwell's equation and electromagnetic waves. Vector notation and vector mathematics are introduced early in the text. The author has prepared this integrated, undergraduate text to provide an understanding of the principles of electric and magnetic fields as a background for the rapidly developing areas of energy conversion. As such, it offers topics not usually included in standard electromagnetic texts, for example, the general features of energy storage, energy flow and energy transfer.

QUALITATIVE ANALYSIS, by Therald Moeller. 550 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $6.50.

The author, a professor of inorganic chemistry at the University of Illinois, has prepared this text as an introduction to equilibrium and solution chemistry. Rather than emphasize either the laboratory approach or solution theory, he has combined the two to give the student a more balanced understanding of qualitative analysis. The book covers the majority of the common anions and cations.

CALCULUS FOR ELECTRONICS, by A. E. Richmond. 407 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $8.25 (text edition available at $6.00).

The author, a registered professional engineer, has spent 25 years in the electronics field as technician, engineer and teacher. The present work reflects this broad background by combining the fundamentals of electronics and the processes of calculus in a realistic, practical approach to the study of electronic equipment systems. In addition to the clear exposition of fundamentals, special topics are covered, including the complex exponential representation of a physical sinusoid and Taylor's expansions of functions of two independent variables. Over 400 problems (with answers to the odd-numbered) are included. Prerequisite for understanding the book is a knowledge of algebra and trigonometry, which should appeal to electronics technicians, radio and TV engineers and electricians.

TELEVISION IN SCIENCE AND INDUSTRY, by V. K. Zworykin, E. G. Ramberg and L. E. Flory. 300 pages, illustrations, 6 × 9 in. New York, John Wiley & Sons, Inc., 1958. Price, $10.00.

Treating television as a means of extending human vision to inaccessible places, the authors discuss the many possibilities of closed-circuit television. Covering such applications as safety, economy in personnel, inspection, data transmission, biological research, medical practice, protection of property and military uses, the first portion of the book will be a fascinating revelation to those who think of television only in terms of home entertainment. One chapter describes in detail the equipment for closed-circuit TV—the camera, circuitry, receiver, and transmission of signals. In this chapter are discussed the use of transistors and miniature tubes, color and stereo apparatus, and the television microscope. A lengthy chapter on the current applications in industry, research, medical science, education, commerce and defense precedes a brief, closing chapter on forecasts for the future. Numerous up-to-date references follow each chapter in this excellent book.

INTRODUCTION TO ELECTROMAGNETIC ENGINEERING, by Roger F. Harrington. 312 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $8.00.

This work is designed for the preliminary study of electromagnetic theory. It introduces field theory, as an extension of circuit theory, obtaining Maxwell's equations early in the text. The approach to mathematics is rather from an engineer's viewpoint than that of a mathematician's. Abstract math has

been replaced wherever possible in favor of physical concepts. The MKSC systems of units, standard among electrical engineers is used throughout, as well as the practical employment of current engineering nomenclature and notation. The last chapters are devoted to the theory of vector analysis and the study of time-varying field theory. Material requiring continual reference is summarized in the appendixes.

FUNDAMENTALS OF HIGH POLYMERS, by O. A. Battista. 140 pages, illustrations, 6 × 9 in. New York, Reinhold Publishing Corp., 1958. Price, $5.50.

A condensed, fundamental text covering the field of high polymer science, written especially for those desiring a basic knowledge of this recently established industry. Chapters dealing with the chemistry of natural and man-made high polymer products and the formation of useful macromolecular products are included. There is also an extensive bibliography. This book serves as a handy reference for chemists, engineers, physicists or biologists engaged in academic research and industrial high polymer technology.

RESEARCH MANAGEMENT, Vol. I, No. 1. 64 pages, diagrams, 6 × 9 in. New York and London, Interscience Publishers, 1958. Price, $7.50 per year (quarterly).

The first issue of this new quarterly devoted to the increasingly important field of research management contains four papers by important research men in the United States. Published by Interscience Publishers in cooperation with the Industrial Research Institute, the new journal is intended to encourage improved industrial research management and to develop an understanding of research as a vital force in our civilization. The broad problems which will be discussed in this quarterly will include organization and functional operation of research departments; design and construction of laboratories; personnel selection, training, compensation and maintenance of morale; creative thinking; project selection; scheduling and control; and the integration of research into the company's operations.

MOTION AND TIME STUDY APPLICATIONS, by Ralph M. Barnes. 188 pages, plates, 8½ × 11 in. New York, John Wiley & Sons, Inc., 1958. Price, $3.50 (paper).

Prepared as a reference manual and work book, this volume can be used in college courses and in industrial training programs. It is a useful supplement to the author's "Motion and Time Study," in that it includes case material and research data lacking in the original work. An interesting feature is the inclusion of two large plates showing two sheets of motion picture film enlargements for use in the practice of micromotion analysis.

CURRENT TOPICS

New Method of Identifying Disaster Victims.—A new means for instant identification of war or disaster victims through electronic computer handling of skull measurements has been reported by Dr. Viken Sassouni, an assistant professor of physical anthropology in the University of Pennsylvania's Graduate School of Medicine.

Taking eight different cranial and facial dimensions from standard X-ray negatives, Sassouni positively identified a simulated victim from among 498 suspects in just over two minutes. It took but ten seconds of that time for the Univac system in the Computing Center at the University to make its correct selection of the "victim's" X-ray after the coded dimensions from a corresponding X-ray film were fed into the machine.

While details as to application of the technique remain to be worked out, Sassouni's method was immediately seen as a potential major aid to military, police, insurance and disaster relief organizations needing to identify the dead. There was speculation that it may be the greatest advance in such identification work since the technique of fingerprinting was perfected. Furthermore, since the human skull does not deteriorate, the new method could assure identification of many victims in the event of atomic bombing—when identifying personnel might not be able to approach the disaster scene for months afterward.

Dr. Sassouni collected 500 pairs of frontal X-rays and then developed eight measurements which are highly individual. He and his brother (who works at the Remington Rand center in Philadelphia) coded these figures for all 500 X-rays into the "memory unit" of the Univac system.

When a participating faculty member chose a particular foot-square negative from the simulated after-death file, Dr. Sassouni made eight quick measurements. He fed the coded results into the Univac. The computer promptly delivered directions for finding the corresponding X-ray picture in the before-death file. The entire procedure took two minutes and 16 seconds.

The vital measurements in the roentgenographic cephalometry process of Sassouni's are: bigonial breadth, mastoid to apex height, bimaxillary breadth, bizygomatic breadth, maximum cranial breadth, sinus breadth, incision height and total facial height.

Dr. Sassouni stressed that further thinking must be done before the method can be widely useful. He said his findings were simply a further step in X-ray analysis experimented with by fellow scientists for the past 25 years. His University colleagues believed that Sassouni's adaptation of X-ray measurements to computer handling, and to positive identification of individuals, was a new first for Pennsylvania—where the giant electronic computer itself was invented a dozen years ago.

The dentist-inventor said he was also proposing a portable picture-taking unit which could X-ray the victim's skull at a standard angle under any conditions. Technicians could be trained in a brief time to read measurements properly from the X-

rays. Thus a catalogue of personal measurements could be built up at a central office—just as the F. B. I. has a massive fingerprint file. Coded equivalents could be telegraphed from the field, and proper identification made at the center and relayed back to field workers at once.

Hydrothermal Process Produces Synthetic Sapphires.—A new hydrothermal technique for growing cultured sapphire crystals has recently been announced by R. A. Laudise and A. A. Ballman, of the Bell Telephone Laboratories. Large sapphires, up to ¾ in. square by ¼ in. thick, have been grown by this technique. The crystals produced are essentially strain-free.

For optical purposes, sapphires grown in this manner should be superior to crystals grown by other available processes, because of their freedom from strain. Since sapphire transmits light well out into the infrared, and far into the vacuum ultraviolet regions, it is a very useful material for such optical components as lenses, prisms, windows, and lamp envelopes.

Aluminum oxide, which is normally considered insoluble in water, can be dissolved and recrystallized from aqueous solutions if the temperature and pressure used are high enough. This is the basis for the hydrothermal process, which employs high temperature–high pressure conditions to increase the solubility of the compound to a point where it can be crystallized on a seed. For growing sapphires, pressures of the order of 20 to 50,000 psi., and temperatures of about 395°C. and higher are used. Rate of growth varies with conditions, but typically it ranges from 0.002 to 0.010 in. per day. Even greater growth rates appear feasible.

Sapphire is known as the corundum modification of aluminum oxide. It is extremely hard, and has applications in many industrial fields, other than in optical applications. Corundum is widely used as an abrasive.

If the nutrient solution is "doped" with small amounts of a chromate, the resultant crystal is synthetic ruby, with the color a function of the amount of chromate added. This chromium-doped crystal has possible applications as a solid state maser.

Different reagents have been tried in the growing process, including aluminum hydroxide and several forms of aluminum oxide. When aluminum hydroxide is used at the proper temperature and pressure, it is all converted after a few days to tiny corundum crystals, instead of being deposited on the major seed crystal. This technique might be of value in making corundum for the abrasive trade.

Computer for Repetitive Business Operations.—A new concept in computers that offers everyday businesses the advantages of computer techniques on frequent and repetitive computational problems was introduced by Burroughs Corporation at the NOMA Business Show in Chicago.

Called the Series F2000 Computer, this newest addition to the Burroughs data processing family represents a highly practical investment for business firms, banks and governmental agencies because of its simplicity of operation and high degree of reliability—along with low original and maintenance costs.

Equipped with mechanical plugboard program selector, the new computer provides greater programming flexibility than in some of the more expensive varieties. Up to 19 memory units may be used for data storage of varying factors or totals with the

Series F computer. Among the data storage features of the new machine are multiplier, multiplicand and product accumulator storage.

Four or more routines may be programmed on each interchangeable mechanical plugboard—and each program directs the computer through all of its automatic operations. All mathematical functions—sub-totalling, totalling, carriage movement, printing and read-out—from any one of the 19 memory storage units may be performed automatically. A full adding machine type keyboard is used for the input of the Series F computer. Among the many applications for which the machine is practical are payroll computation and check preparation, tax billing and loan scheduling.

New Ethylenic Coated Fabric.—A new, opaque coated fabric which remains pliable in both hot and cold temperatures, possesses an unusually long flex life, and has excellent resistance to edge wear is now being produced by the Du Pont Company's Fabrics Division.

Designed primarily as a heavy-duty camera bellows material, Du Pont "Armalon" ethylenic resin coated fabric is expected to have many potential applications in aircraft and industry. Among these are: dark room curtaining, pull shades in military aircraft, protective covers for flight instruments, drawing-board covers, dust covers for industrial equipment, particularly where a high degree of mar resistance is required. It is unaffected by contact with lacquered surfaces.

"Armalon" is available in black and gray and is produced in two gauges—0.007 in. and 0.010 in. The material comes in 36-in. and 42-in. widths. A sample swatch may be obtained by writing to Room 7031-D, Du Pont Company, Wilmington 98, Delaware.

Ferromagnetism Without Ferromagnetic Elements.—Ferromagnetism at low temperatures has been discovered in an intermetallic compound which does not contain any of the commonly known ferromagnetic elements. This discovery, made by B. T. Matthias at Bell Telephone Laboratories, was achieved while studying the properties of a zirconium zinc compound ($ZrZn_2$) at low temperature.

The behavior of zirconium zinc indicates that ferromagnetic and perhaps antiferromagnetic compounds may be formed by the combination of many more metals than had been supposed until now. Thus, the discovery opens up an entirely new field for scientific exploration.

Heretofore, no ferromagnetic intermetallic compound was known that did not contain any ferromagnetic elements—iron, cobalt, nickel, chromium, manganese, or a few rare earth metals. This has led to the tacit assumption that no ferromagnetism could occur in an intermetallic compound unless it contained at least one strongly paramagnetic element.

Zirconium zinc becomes ferromagnetic below 35°K. Its ferromagnetic characteristics are similar to and of the same order of magnitude as those exhibited by conventional ferrites at room temperatures. Discovery of these properties provides scientists with a powerful new tool for the study of the fundamental characteristics of magnetism and magnetic materials.

Research Reactor Shuts off Automatically.—A nuclear research reactor, known as KEWB, increased its power output from zero to 530,000 thermal kilowatts in less than a second, it has been announced by Atom-

ics International, a division of North American Aviation, Inc. After the power increase, the reactor shut itself off without sustaining any damage.

This rate of power increase is the largest known to have been experienced safely by a reactor and amounted to tripling the power every two-thousandths of a second. The power surge occurred in a special test to study the behavior of an aqueous homogeneous reactor if operating control were lost. In this test, held April 14, KEWB was subjected to severe operating conditions when far more fissionable uranium than required for normal operation was suddenly made available for a chain reaction by the removal of the control rod.

KEWB (Kinetic Experiment on Water Boilers) was designed and built for the Atomic Energy Commission by Atomics International to conduct reactor safety experiments under scientifically controlled conditions. It is a 50 kilowatt aqueous homogeneous research reactor fueled with a solution of uranium-235. Atomics International operates the reactor, located in the Santa Susana Mountains northwest of Los Angeles, as part of the Commission's reactor safety program.

The special test demonstrated the inherent ability of the reactor to shut itself down without the use of auxiliary control rods or devices when an abnormal operating condition occurs. The power surge was so brief that the temperature of the fuel solution was raised only slightly, not enough to cause it to boil. Only moderate pressure lasting for one-thousandth of a second was generated within the reactor vessel. The power was prevented from rising further by the sudden expulsion of fuel solution into an overflow chamber which prevented the nuclear chain reaction from continuing. The expulsion of the fuel solution, in turn, was caused by the generation of gas within the fuel, a characteristic of solution-type reactors under high power operation.

Other homogeneous solution-type reactors are being used for research in physics, chemistry, medicine, metallurgy and other scientific and industrial fields, as well as for training nuclear scientists. Atomics International has built several reactors of this type which are now operating in the United States and abroad.

Radioactive Wastes in Sewage Treatment.—Radioactive waste products of nuclear power reactors could be used in water intake and sewage treatment plants to effect a 99 per cent reduction in harmful bacteria and at the same time lessen plant operation costs, University of Michigan research indicates.

Significantly, too, the technique would take beneficial advantage of the radioactive waste products which now have to be stored beneath the ground or elsewhere until they lose their dangerous radioactivity. Finding uses for such products presently is one of the most challenging problems concerned with the operation of nuclear power reactors.

While actual details would have to be worked out by sanitary engineers, the U-M scientists say harmful bacteria could be killed by running water or sewage past a core of radioactive waste products. This core would act similar to a huge X-ray machine, and because of the nature of the radioactive waste products and their installation, the water and material in the water would not become radioactive.

The core would remain active for several years before a replacement would be necessary. The use of this

material could effect a saving in the cost of treatment of some waters, since there is some evidence that similar amounts of water treatment chemicals would be required.

Basic cost of the radioactive "waste products" would be in the handling of them between the source and the treatment center, but probably a regular schedule could be worked out to keep this at a minimum.

The research was done by Gerald M. Ridenour, professor of public health engineering, and Edward H. Armbruster, president lecturer in public health engineering. It was conducted under a grant from the Michigan Memorial-Phoenix Project and the National Sanitation Foundation.

Multiplex Reduction Printer.—A new multiplex reduction printer, suitable for preparing multiplex diapositives from aerial photography taken with either a 6-in. Metrogon or a 6-in. distortion free lens, has been developed by the Corps of Engineers' Research and Development Laboratories, Fort Belvoir, Va.

The new reduction printer is basically distortion-free but may be converted to correct for nominal Metrogon distortion by the addition of an aspheric corrector plate to the system. The aspheric plate is installed by simply removing the film pressure plate and inserting the aspheric plate in its place. Since the two plates are not the same thickness and the change affects the focus of the printer, a spacer ring is provided that must be installed in the head section to correct the optical distance.

The resolving power of the new printer, both with and without the aspheric corrector plate, is superior to that previously obtained with reduction printers designed to correct for Metrogon distortion only.

The new multiplex reduction printer solves the problem of converting to the use of distortion free photography for mapping while many mapping cameras with Metrogon lenses are still being used. When and if the Metrogon cameras become completely obsolete, the aspheric plates may simply be omitted.

Low-Noise Amplifier Uses Semiconductor Diodes.—A new device which has exciting possibilities as a low-noise u.h.f. and microwave amplifier is under development at Bell Telephone Laboratories. Although still in the experimental stage, preliminary results indicate that this device, which uses semiconductor diodes as the active elements, can improve the performance of many types of microwave receivers. It is relatively simple to construct and operate, and shows prospects of having a long life.

Noise is a major problem in the amplification of weak microwave signals. Commercially available amplifiers and converters add a considerable amount of noise to the incoming signal, thus decreasing the sensitivity of the receiving equipment. A major reduction in this added noise can significantly improve the performance of radio receivers such as those used in radar, radio astronomy, over-the-horizon radio relay and u.h.f. television systems. The diode amplifier holds promise of providing such a noise reduction.

This amplifier is one of a family of devices known as variable reactance amplifiers in which a variable reactance, or "varactor," serves as the active component. In the present device, the variable reactance is provided by a semiconductor diode (varactor diode) whose capacitance varies with the applied voltage. As with

other varactor amplifiers, the applied voltage is derived from a high-frequency pump signal. This signal causes the diode to function as a time-varying capacitance and supplies the energy which is necessary to produce amplification. Low-noise amplification using varactor diodes was predicted by A. Uhlir, Jr. of Bell Laboratories.

Amplification at 6000 megacycles was first demonstrated by M. E. Hines and H. E. Elder of Bell Laboratories. At the same frequency, G. F. Herrmann and M. Uenohara later obtained a bandwidth of 8 megacycles with a noise figure of 5 to 6 db. Gain was 18 db. and the pump signal 12,000 megacycles. Gain can be traded for additional bandwidth if desired, and vice versa.

A traveling-wave amplifier configuration using arrays of several diodes shows promise of providing bandwidths of 25 per cent or more in the u.h.f. region. Using four stages with the special diodes in such an array, R. S. Engelbrecht of Bell Laboratories has obtained a bandwidth of 100 megacycles at a 400-megacycle signal frequency, with a pump frequency of 900 megacycles and a pump power of 10 milliwatts. This experimental amplifier has a gain of 10 db. and a noise figure of only 3½ db.

A single type of diode can be used to make an amplifier for any desired frequency from the high microwave region down to d.c. The noise performance improves rapidly as the frequency decreases from microwaves down into the u.h.f. region, thus making such an amplifier potentially useful for u.h.f. television receivers.

Interesting aspects of the diode amplifier are its simplicity and potial reliability. Major components are the proper waveguide structures, the diode itself, and a suitable pump signal source. It appears that these components can be assembled to provide a relatively inexpensive device. No refrigeration is required, and no magnetic fields are necessary. The low-noise characteristics are realizable at room temperatures.

Although the variable capacity effect is present in commercial diodes, Bell Laboratories' scientists have developed, under a Signal Corps contract, special diffused silicon diodes to maximize this effect. Series resistance, which could be a source of noise, is minimized in these diodes. Units fabricated by N. G. Cranna of the Laboratories have an active diameter of about 0.002 in.

The development of the varactor diode amplifier holds promise of providing a whole new family of low-noise amplifiers for the u.h.f. and microwave frequency ranges. These devices will complement the semiconductor diode up-converters recently announced by Bell Laboratories, which can also provide low-noise amplification.

Closed Circuit TV Used in Training Nuclear Plant Operators.—As the Army's first facility for training nuclear power plant operators, the Army Reactor at Fort Belvoir includes the latest features for giving the best instruction to the most trainees in the minimum time. One of these features is a closed circuit television system designed to allow a classroom full of students to observe intricate maintenance functions or radioactive equipment operation in other parts of the plant.

Designed and built by the Dage Corporation for Alco Products, Incorporated, the Atomic Energy Commission's prime contractor on the Army Package Power Reactor, the TV monitor system includes a camera pick-up

which can be located inside the plant's "Vapor Container," in which the reactor and radioactive, high pressure water system is located.

During the infrequent intervals when the APPR-1 is shut down for re-fueling, the TV system will be used for trainees to observe the delicate operations required by the small, skilled maintenance crew in the restricted work space inside the "vapor container." Thus, trainees can become familiar with the procedure without distracting or interfering with the crew's operations.

Outlets are also available throughout the APPR-1 so that the pick-up camera can monitor any operation in the plant and permit large groups to observe maintenance procedures in any restricted area.

"Atomic" Amplifier.—Philco Research has announced development of a special type of gas Maser, also known as an "atomic" amplifier. Allen C. Munster, Director of Research for the Government and Industrial Division, said the Maser study is being conducted for the Army Signal Engineering Laboratory, Fort Monmouth, N. J. and is an important part of the Army's pioneering work in this field.

The Philco Maser is the first "atomic" amplifier to demonstrate unilateral gain of microwave energy by means of two electrically isolated cavities connected by a beam of neutral ammonia gas molecules. Energy is transferred from one electromagnetically isolated point to another by the flow of these molecules. As the oscillating molecules pass through the second cavity, electromagnetic radiation is developed and it appears in the output waveguide. Thus is achieved unilateral amplification without aid of electronic effects.

Just as electronics has achieved its greatest versatility and reliability through the inherently unilateral amplifying property of vacuum tubes and transistors, the new unilateral Maser promises equally important applications for atomic amplifiers.

Masers have excited the electronics industry because they remove the noise inherent in conventional electronic devices. They can provide amplification approaching the theoretical limit of no extraneous noise. It is the noise in an electronic system that produces the static on radio and the white fuzz (snow) on the television screen.

The Philco Maser also has demonstrated the predicted physical phenomenon of prestimulation of molecules in an excited state. Prestimulation in a two-cavity device promises to provide a greater degree of freedom for controlling the gain-bandwidth properties of an "atomic" amplifier.

Masers are still on the threshold of a scientific frontier, but soon new systems will be evolved. Their successful operation will depend either upon very low noise amplification or very high oscillator stability which represents a dramatic technological breakthrough for electronic components.

Telephony via the Moon.—In a few years it may be possible to pick up the telephone and talk to someone halfway around the earth by using the moon as a reflector. Research performed at the University of Michigan indicates that very short wave radio or radar signals, which can be used in voice transmission, bounce off the moon without appreciable loss in quality.

This means the curvature of the earth no longer need be a factor in limiting communication distance—a radar signal has to travel in a straight

line—so long as both transmitting and receiving equipment can "see" the moon.

The research was conducted under a contract the U-M Engineering Research Institute had with the Rome Air Development Center of the U. S. Air Force and was done by Prof. Keeve M. Siegel and Thomas B. A. Senior, research associate.

It has been known for several years that voice communication using the moon as a reflector was possible when long wave length signals were utilized. But the University investigators have found that very short wave lengths of about an inch give such improvement in clarity that it now is commercially feasible to set up a world-wide network of stations that would use the moon in this way.

Another significant aspect of the work is that careful analysis and measurement of radar signals reflected from the moon does yield additional data on the composition of the moon's surface.

Such data would be expected to help fill present gaps in lunar knowledge outlined in the recent recommendations on scientific research in outer space prepared by the technical panel on the Earth Satellite Program of the U. S. National Committee for the International Geophysical Year.

Information already obtained in this way indicates that the moon's surface is composed of a sandy-like material. This corroborates data from astronomers and others studying the moon.

The U-M scientists met success in developing the reflection theory when, in checking reports by other researchers throughout the nation, they found certain incorrect conclusions had been drawn. One of these was the assumption that, because of the jagged mountains and craters which are known to exist on the moon, the moon would appear rough to a radar signal.

But a short wave signal only requires an area of a few square feet to be reflected. The moon has such areas that are smooth and as a result the radar signals are reflected without a loss in quality.

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